A Reinforcement-Learning Framework for Symmetric Reordering in Sparse Cholesky Factorization

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How I will attempt to solve a hard problem and then try to understand it

aka: Solve first, then think

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From Interior-Point method [Wright, 1997]:

- Solve (sparse) **symmetric linear system**

$$\Phi \Delta y = \xi, \quad \text{with } \Phi = (A \Theta A^T) \succ 0$$

- Compute (sparse) **Cholesky factorization**

$$\Phi = L \times L^T, \quad \text{with } L \text{ lower triangular}$$

- **BUT** even if Φ is sparse, L may not be!

$\Rightarrow$ re-order rows and columns of Φ before computing L

Ordering matters

$$\Phi = \begin{pmatrix} \times & & & & \times \\ & \times & & & \times \\ & & \times & & \times \\ & & & \times & \times \\ \times & \times & \times & \times & \times \end{pmatrix} \rightarrow L = \begin{pmatrix} \times & & & & \\ & \times & & & \\ & & \times & & \\ & & & \times & \\ \times & \times & \times & \times & \times \end{pmatrix}$$ **Good**

Ordering matters

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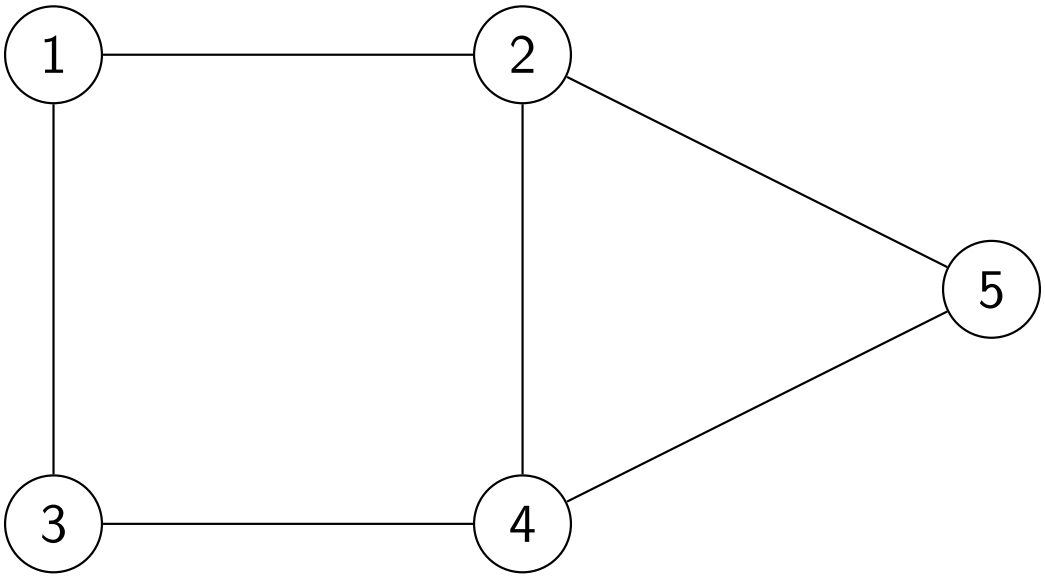
$$\Phi = \begin{pmatrix} \times & \times & \times & \times & \times \\ \times & \times & & & \\ \times & & \times & & \\ \times & & & \times & \\ \times & & & & \times \end{pmatrix} \rightarrow L = \begin{pmatrix} \times & & & & \\ \times & \times & & & \\ \times & + & \times & & \\ \times & + & + & \times & \\ \times & + & + & + & \times \end{pmatrix} \quad \textbf{Bad}$$

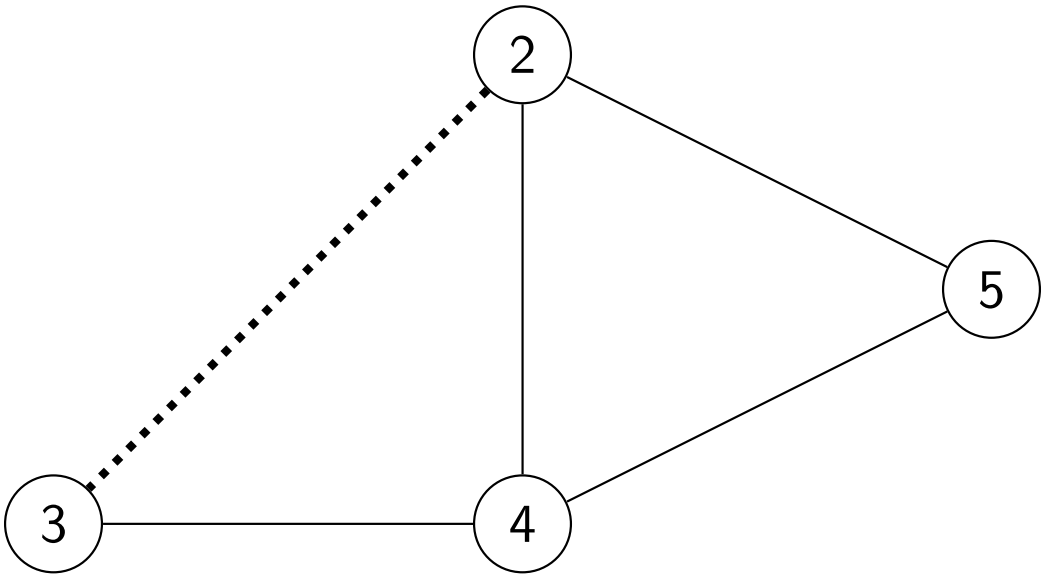
Figure: $\times$ are non-zeros elements of Φ ; $+$ are fill-in (zero in Φ but not in L)

Finding a good ordering reduces to a **Graph elimination problem**:

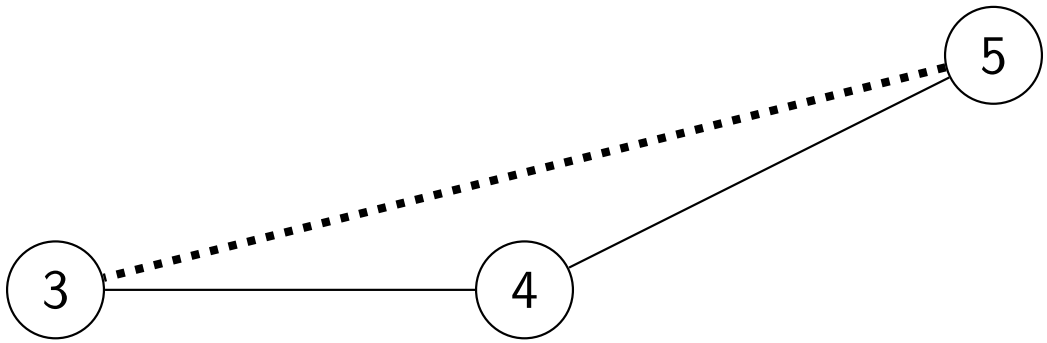
- Input is undirected graph G
- Remove a vertex $v \in G$ and add edges so that neighbours of v form a clique. Repeat until G has no more vertices.
- Goal: find an elimination sequence that minimizes the total number of new edges.



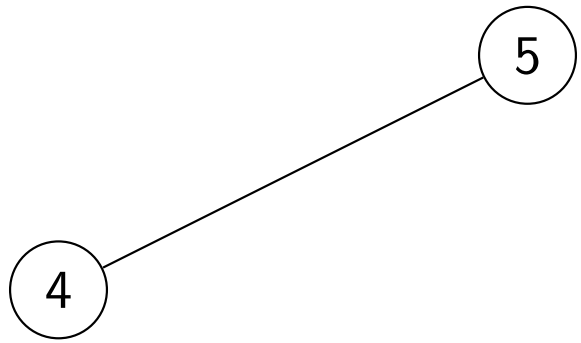
Elimination order:



Elimination order: 1,



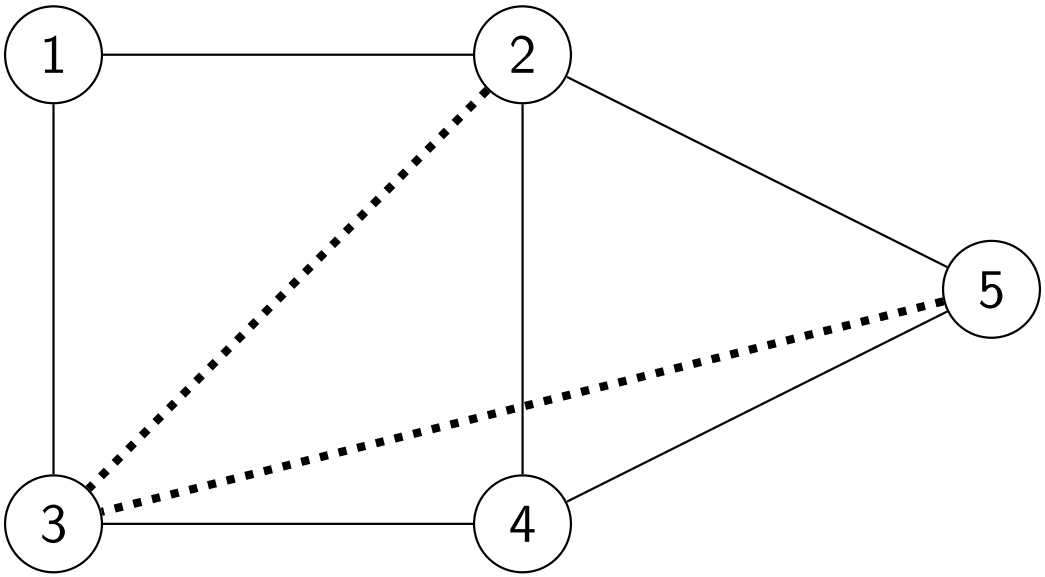
Elimination order: $1, 2,$



Elimination order: $1, 2, 3,$

Elimination order: $1, 2, 3, 4,$

Elimination order: $1, 2, 3, 4, 5$



Elimination order: $1, 2, 3, 4, 5 \Rightarrow 2$ edges added

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Reinforcement Learning

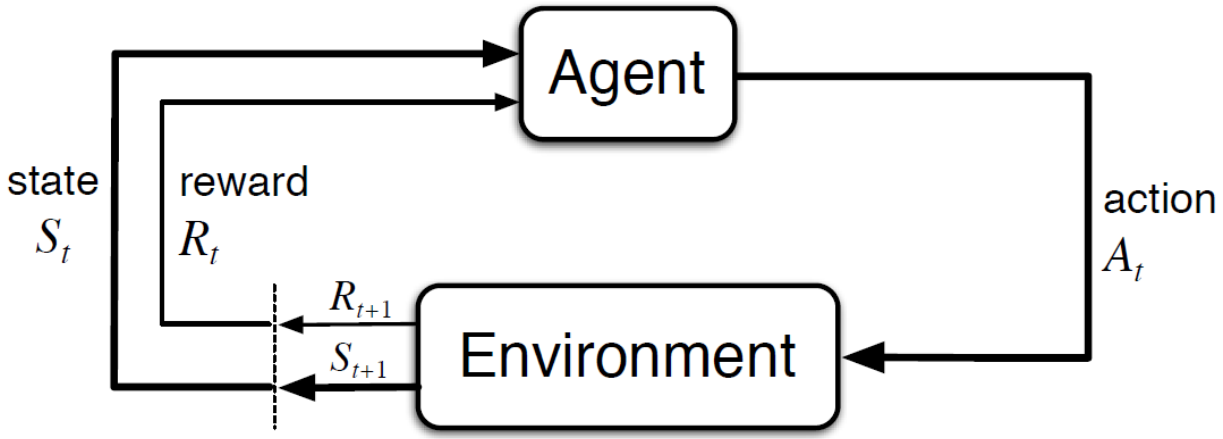
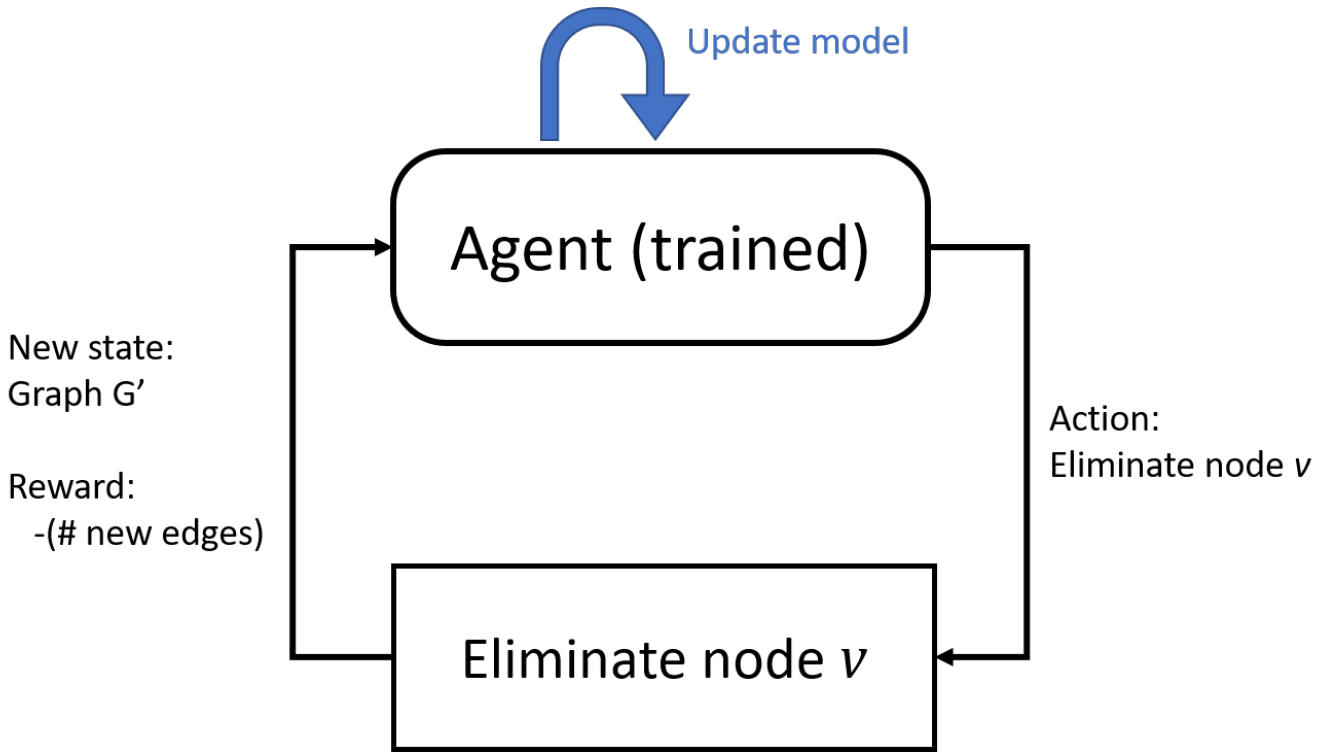


Figure 3.1: The agent–environment interaction in reinforcement learning.

- Maximize long-term cumulative reward
- **Learn a policy** (i.e. which action do I take, given the current state)
- **Theorem:** *there exists an optimal greedy policy*

See *Reinforcement Learning*, Sutton, Barto 2017

Goal: use RL to learn greedy heuristics



Can we learn heuristics...

- ...at all? (likely, yes)
- ...that perform as good as existing ones? (likely, yes)
- ...that also run as fast as existing ones? (likely, no)
- ...for other objectives? (hopefully, yes)

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What can we learn from those (learned) heuristics?

- New heuristics
- Underlying problem structure (by looking at decision rules)

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Take away

- Consider a (combinatorial) optimization problem
- Attempt to solve it using an RL model
- Use that model to infer further knowledge, e.g.: decomposition? parallelism?


Wright, S. (1997).
Primal-Dual Interior-Point Methods.
Society for Industrial and Applied Mathematics.