Long time and large scale behaviour of a few collisional dynamics

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This Ph.D. thesis presents the study of three particle systems, respectively arising from nonequilibrium statistical mechanics, stochastic portfolio theory, and the analysis of systems of nonlinear partial differential equations. The corresponding dynamics are either stochastic or deterministic, and have the peculiarity of interacting only through collisions between the particles. Motivated by quantitative questions regarding the underlying models, various aspects of the long time and large scale behaviour of these systems are addressed.

The first part of the thesis is dedicated to the *Complete Exchange Model*, which was introduced by physicists as a lattice Hamiltonian model of heat transfer in porous media. A numerical study is first performed to show that, in dimension 1, when the model is put in contact with heat baths at different temperatures, the temperature profile along the chain does not obey Fourier's law. This advocates for a detailed understanding of nonequilibrium steady states of the dynamics, which is unfortunately a very difficult task from the mathematical point of view. The main contribution of this part of the thesis is the introduction, based on a stochastic billiard interpretation and making an intense use of Markov renewal theory, of a rigorous setting to describe the ergodicty of the model. For an elementary instance of two thermalised particles, a complete convergence result is obtained at thermal equilibrium, and partial progresses are made out of equilibrium.

The second part of the thesis addresses *systems of diffusions interacting through their rank*, that is Brownian particles evolving on the real line with rank-dependent drift and diffusion coefficients. Such systems were introduced in stochastic portfolio theory as first-order approximations of stable equity markets. When the drift and diffusion coefficients describe mean-field interactions between stocks, a propagation of chaos result is first obtained: in the limit of an infinite market, the stocks are proven to behave like independent copies of a nonlinear (in McKean's sense) diffusion process. This result provides a hydrodynamic description of the distribution of the capital among the companies on the market. Adapting recent techniques from optimal transport and the study of McKean-Vlasov models, the long time behaviour of the nonlinear process is then described, which sheds light on a phase transition in the stationary distribution of the capital: depending on the parameters of the model, either the capital is aggregated between a few companies, or it is well-spread between all companies, according to an explicit capital density.

The last part of the thesis contains the derivation and study of the *Multitype Sticky Particle Dynamics*, which is introduced in the purpose of constructing global weak solutions to one-dimensional diagonal hyperbolic systems. In general, (strictly) hyperbolic systems are known to possess global weak solutions only under a smallness assumption on the total variation of the data. The construction provided here overcomes this restriction, but in turn assumes that the data are monotonic. It relies on the introduction of a multitype version of the sticky particle dynamics, which is a classical model in astrophysics as well as in the study of pressureless gases. Each type of particle is associated with one conserved quantity of the hyperbolic system. Although the evolution of the particle system is entirely deterministic, a probabilistic analysis of the empirical distribution of the particle system allows to construct weak solutions to the system when the number of particles grows to infinity. A uniform stability estimate on the particle system is then derived through a careful analysis of the trajectories of the Multitype Sticky Particle Dynamics, which involves the resolution of a purely combinatoric problem. This leads to original Wasserstein stability estimates on the solutions of the initial hyperbolic system, which are also proven to be semigroup solutions.