Algorithms For Two-Time-Scales Stochastic Optimization With Applications To Long Term Management Of Energy Storage

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Local renewable energies are spreading

To lower CO_2 emissions from our electricity generation

We tend to consume energy where it is produced

But they require a storage that has to be managed

When intermittent renewables generation does not match demand
we rely on fossil fuels

Storage cleans our electricity generation
as long as we optimize its management to make it sustainable

Real problems adressed by the optimization team at Efficacity

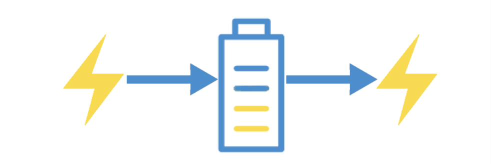
Our team solves energy management problems
for the energy transition of cities with our industrial partners

RATP case study

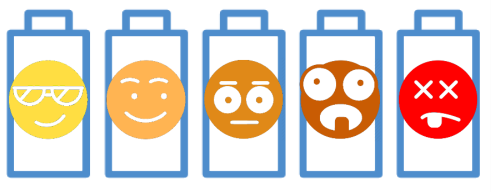
VINCI Energies case study

We optimize storage management on multiple time scales

to store/consume clean energy at the right minutes of the day

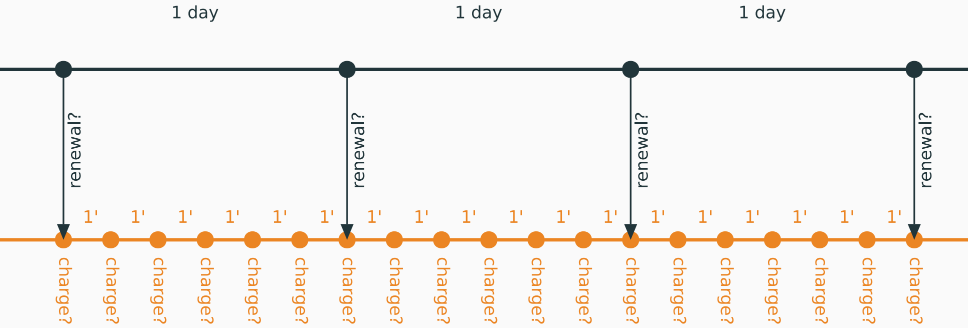


and to ensure a sustainable battery life lasting many years



We tackle battery control problems on two time scales

We will decompose the scales



Two time scales stochastic optimal control problem

$$\min_{\boldsymbol{X}_{0:D+1}, \boldsymbol{U}_{0:D}} \mathbb{E}\left[\sum_{d=0}^{D} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + K(\boldsymbol{X}_{D+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d)$$

$$\boldsymbol{U}_d = (\boldsymbol{U}_{d,0}, \ldots, \boldsymbol{U}_{d,m}, \ldots, \boldsymbol{U}_{d,M})$$

$$\boldsymbol{W}_d = (\boldsymbol{W}_{d,0}, \ldots, \boldsymbol{W}_{d,m}, \ldots, \boldsymbol{W}_{d,M})$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma\big(\boldsymbol{W}_{d',m'}; (d', m') \leq (d, m)\big)$$

We have a non standard problem

- with daily time steps
- but a non anticipativity constraint every minute

Next to come: outline of the talk

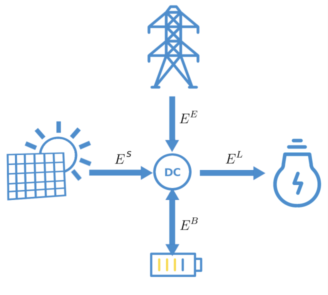
I. Illustration with an energy storage management application

II. Two algorithms for two-time scales stochastic optimization

III. Numerical results for a house with solar panels and batteries

Application to energy storage management

Physical model: a home with load, solar panels and storage



- Two time scales uncertainties
  - $\boldsymbol{E}^L_{d,m}$: Uncertain demand
  - $\boldsymbol{E}^S_{d,m}$: Uncertain solar electricity
  - $\boldsymbol{P}^b_d$: Uncertain storage cost
- Two time scales controls
  - $\boldsymbol{E}^E_{d,m}$: National grid import
  - $\boldsymbol{E}^B_{d,m}$: Storage charge/discharge
  - $\boldsymbol{R}_d$: Storage renewal
- Two time scales states
  - $\boldsymbol{B}_{d,m}$: Storage state of charge
  - $\boldsymbol{H}_{d,m}$: Storage health
  - $\boldsymbol{C}_d$: Storage capacity
- Balance equation:

$$\boldsymbol{E}^E_{d,m} + \boldsymbol{E}^S_{d,m} = \boldsymbol{E}^B_{d,m} + \boldsymbol{E}^L_{d,m}$$

- Battery dynamic:

$$\boldsymbol{B}_{d,m+1} = \boldsymbol{B}_{d,m} - \frac{1}{\rho_d}\boldsymbol{E}^{B-}_{d,m} + \frac{1}{\rho_d}\rho_c\boldsymbol{E}^{B+}_{d,m}$$

New dynamics: aging and renewal model

- At the end of every day d, we can buy a new battery at cost $\boldsymbol{P}_d^b \times \boldsymbol{R}_d$

$$\text{Storage capacity: } \boldsymbol{C}_{d+1} = \begin{cases} \boldsymbol{R}_d \,, & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{C}_d \,, & \text{otherwise} \end{cases}$$

example: a Tesla Powerwall 2 with 14 kWh costs $430 \times 14 = 6020$ €

- A new battery can make a maximum number of cycles $N_c(\boldsymbol{R}_d)$:

$$\text{Storage health: } \boldsymbol{H}_{d+1,0} = \begin{cases} 2 \times N_c(\boldsymbol{R}_d) \times \boldsymbol{R}_d \,, & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{H}_{d,M} \,, & \text{otherwise} \end{cases}$$

$\boldsymbol{H}_{d,m}$ is the amount of exchangeable energy day d, minute m

$$\boldsymbol{H}_{d,m+1} = \boldsymbol{H}_{d,m} - \frac{1}{\rho_d} \boldsymbol{E}_{d,m}^{B-} - \rho_c \boldsymbol{E}_{d,m}^{B+}$$

example: a Tesla Powerwall 2 can make 3200 cycles or exchange 90 MWh

- A new battery is empty

$$\text{Storage state of charge: } \boldsymbol{B}_{d+1,0} = \begin{cases} \underline{B} \times \boldsymbol{R}_d \,, & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{B}_{d,M} \,, & \text{otherwise} \end{cases}$$

We build a non standard stochastic optimal control problem

- Objective to be minimized

$$\mathbb{E}\Big[\sum_{d=0}^{D}\Big(\underbrace{\boldsymbol{P}^b_d \times \boldsymbol{R}_d}_{\text{renewal}} + \sum_{m=0}^{M-1}\underbrace{p^e_{d,m}}_{\text{price}} \times \big(\underbrace{\boldsymbol{E}^B_{d,m} + \boldsymbol{E}^L_{d,m+1} - \boldsymbol{E}^S_{d,m+1}}_{\text{national grid energy consumption}}\big)\Big)\Big]$$

- Controls

$$\boldsymbol{U}_d = (\boldsymbol{E}^B_{d,0} \ldots, \boldsymbol{E}^B_{d,m}, \ldots, \boldsymbol{E}^B_{d,M-1}, \boldsymbol{R}_d)$$

- Uncertainties

$$\boldsymbol{W}_d = \left(\begin{pmatrix}\boldsymbol{E}^S_{d,1}\\ \boldsymbol{E}^L_{d,1}\end{pmatrix}, \ldots, \begin{pmatrix}\boldsymbol{E}^S_{d,m}\\ \boldsymbol{E}^L_{d,m}\end{pmatrix}, \ldots, \begin{pmatrix}\boldsymbol{E}^S_{d,M-1}\\ \boldsymbol{E}^L_{d,M-1}\end{pmatrix}, \begin{pmatrix}\boldsymbol{E}^S_{d,M}\\ \boldsymbol{E}^L_{d,M}\\ \boldsymbol{P}^b_d\end{pmatrix}\right)$$

- States and dynamics

$$\boldsymbol{X}_d = \begin{pmatrix}\boldsymbol{C}_d\\ \boldsymbol{B}_{d,0}\\ \boldsymbol{H}_{d,0}\end{pmatrix} \text{ and } \boldsymbol{X}_{d+1} = f_d\big(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d\big)$$

Two time scales stochastic optimal control problem

$$\mathcal{P}: \quad \min_{\boldsymbol{X}_{0:D+1}, \boldsymbol{U}_{0:D}} \mathbb{E}\Big[\sum_{d=0}^{D} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + K(\boldsymbol{X}_{D+1})\Big],$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d),$$

$$\boldsymbol{U}_d = (\boldsymbol{U}_{d,0}, \dots, \boldsymbol{U}_{d,m}, \dots, \boldsymbol{U}_{d,M})$$

$$\boldsymbol{W}_d = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,m}, \dots, \boldsymbol{W}_{d,M})$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma\big(\boldsymbol{W}_{d',m'}; (d',m') \leq (d,m)\big)$$

Two time scales because of the non anticipativity constraint

Information grows every minute!

- Intraday time stages: $M = 24 * 60 = 1440$ minutes
- Daily time stages: $D = 365 * 20 = 7300$ days
- $D \times M = 10,512,000$ stages!

Time decomposition by daily dynamic programming

Daily management when "end of the day" cost is known

On day d assume that we have a final cost $V_{d+1} : \mathbb{X}_{d+1} \to [0, +\infty]$

giving a price to a battery in state $\boldsymbol{X}_{d+1} \in \mathbb{X}_{d+1}$

Solving the intraday problem with a final cost

$$\min_{\boldsymbol{X}_{d+1}, \boldsymbol{U}_d} \mathbb{E}\left[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) + V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)$$

$$\boldsymbol{U}_d = (\boldsymbol{U}_{d,0}, \dots, \boldsymbol{U}_{d,m}, \dots, \boldsymbol{U}_{d,M})$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Gives a minute scale policy for day d that takes into account the future through V_{d+1}, the daily value of energy storage

We write a Bellman equation with daily time blocks

Daily Independence Assumption

$\{\boldsymbol{W}_d\}_{d=0,\dots,D}$ is a sequence of independent random variables

We set $V_{D+1} = K$ and then by backward induction:

$$
\begin{aligned}
V_d(x) = \min_{\boldsymbol{X}_{d+1}, \boldsymbol{U}_d} \; & \mathbb{E}\Big[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) + V_{d+1}(\boldsymbol{X}_{d+1}) \Big] \\
\text{s.t } & \boldsymbol{X}_{d+1} = f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) \\
& \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})
\end{aligned}
$$

where $\boldsymbol{W}_{d,0:m} = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,m})$ = non independent random variables

Proposition

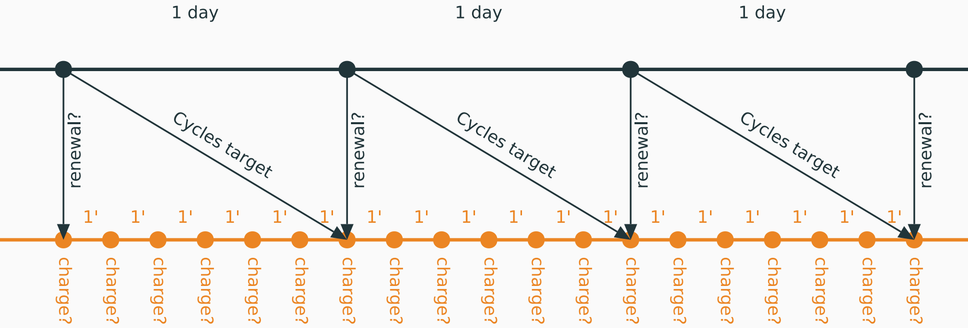
Under Daily Independence Assumption V_0 is the value of problem $\mathcal{P}$

We present two efficient time decomposition algorithms
to compute upper and lower bounds
of the daily value functions

1. **Targets decomposition gives an upper bound**
2. **Weights decomposition gives a lower bound**

Targets decomposition algorithm

Decomposing by sending targets



Stochastic targets decomposition

We introduce the stochastic target intraday problem

$$\phi_{(d,=)}\big(x_d, \boldsymbol{X}_{d+1}\big) = \min_{\boldsymbol{U}_d} \quad \mathbb{E}\Big[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)\Big]$$

$$\text{s.t } f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) = \boldsymbol{X}_{d+1}$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Proposition

Under Daily Independence Assumption, V_d satisfies

$$V_d(x) = \min_{\boldsymbol{X} \in L^0(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{X}_{d+1})} \Big(\phi_{(d,=)}\big(x, \boldsymbol{X}\big) + \mathbb{E}\big[V_{d+1}(\boldsymbol{X})\big]\Big)$$

$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d)$$

Relaxed stochastic targets decomposition

We introduce a relaxed target intraday problem

$$\phi_{(d,\geq)}\big(x_d, \boldsymbol{X}_{d+1}\big) = \min_{\boldsymbol{U}_d} \quad \mathbb{E}\Big[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)\Big]$$

$$\text{s.t } f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) \geq \boldsymbol{X}_{d+1}$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

A relaxed daily value function

$$V_{(d,\geq)}(x) = \min_{\boldsymbol{X} \in L^0(\Omega,\mathcal{F},\mathbb{P};\mathbb{X}_{d+1})} \Big(\phi_{(d,\geq)}\big(x, \boldsymbol{X}\big) + \mathbb{E}\big[V_{(d+1,\geq)}(\boldsymbol{X})\big]\Big)$$

$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d)$$

Because of relaxation $V_{(d,\geq)} \leq V_d$ but $V_{(d,\geq)}$ is hard to compute due to the stochastic targets

Relaxed deterministic targets decomposition

Now we can define value functions with deterministic targets:

$$V_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\phi_{(d,\geq)}\big(x, X\big) + V_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)$$

Monotonicity Assumption

The daily value functions V_d are non-increasing

Theorem

Under Monotonicity Assumption

- $V_{(d,\geq)} = V_d$
- $V_{(d,\geq,\mathbb{X}_{d+1})} \geq V_{(d,\geq)} = V_d$

There are efficient ways to compute the upper bounds $V_{(d,\geq,\mathbb{X}_{d+1})}$

Numerical efficiency of deterministic targets decomposition

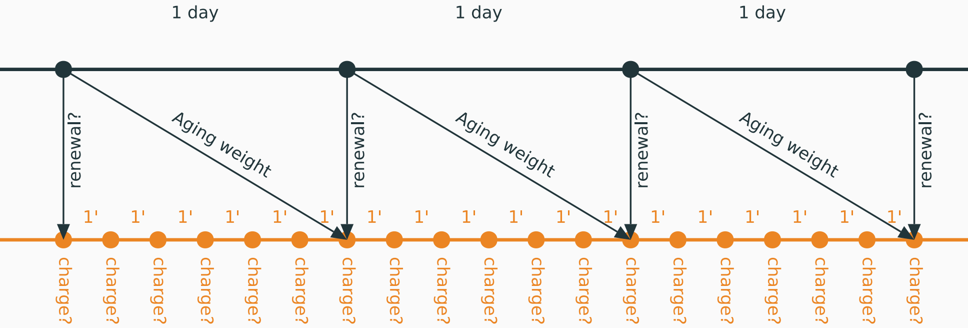
$$\overbrace{V_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X\in\mathbb{X}_{d+1}}\Big(\underbrace{\phi_{(d,\geq)}(x,X)}_{\text{Hard to compute}} + V_{(d+1,\geq,\mathbb{X}_{d+1})}(X)\Big)}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute $\phi_{(d,\geq)}(x,X)$ for each couple (x,X) and each day d but

- We can exploit periodicity of the problem, e.g $\phi_{(d,\geq)} = \phi_{(0,\geq)}$
- In some cases $\phi_{(d,\geq)}(x,X) = \phi_{(d,\geq)}(x-X,0)$
- We can parallelize $\phi_{(d,\geq)}$ computation on day d
- We can use any suitable method to solve the multistage intraday problems $\phi_{(d,\geq)}$ (SDP, scenario tree based SP...)

Weights decomposition algorithm

Decomposing by sending weights



Stochastic weights decomposition

We introduce the dualized intraday problems

$$\psi_{(d,\star)}(x_d, \boldsymbol{\lambda}_{d+1}) = \min_{\boldsymbol{U}_d} \mathbb{E}\Big[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + \langle \boldsymbol{\lambda}_{d+1}, f_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d)\rangle\Big]$$

$$\text{s.t } \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Note that $\psi_{(d,\star)}$ might be simpler than $\phi_{(d,\geq)}$ (state reduction)

Stochastic weights daily value function

$$V_{(d,\star)}(x_d) = \sup_{\boldsymbol{\lambda}_{d+1} \in L^q(\Omega,\mathcal{F},\mathbb{P};\Lambda_{d+1})} \psi_{(d,\star)}(x_d, \boldsymbol{\lambda}_{d+1}) - \Big(\mathbb{E}V_{(d+1,\star)}\Big)^{\star}(\boldsymbol{\lambda}_{d+1})$$

$$\text{s.t } \sigma(\boldsymbol{\lambda}_{d+1}) \subset \sigma(\boldsymbol{X}_{d+1})$$

where $\Big(\mathbb{E}V\Big)^{\star}(\boldsymbol{\lambda}_{d+1}) = \sup_{\boldsymbol{X} \in L^p(\Omega,\mathcal{F},\mathbb{P};\mathbb{X}_{d+1})} \langle \boldsymbol{\lambda}_{d+1}, \boldsymbol{X}\rangle - \mathbb{E}\big[V(\boldsymbol{X})\big]$
is the Fenchel transform of $\mathbb{E}V$

Deterministic weights decomposition

We define value functions with deterministic weights

$$V_{(d,\star,\mathbb{E})}(x_d) = \sup_{\lambda_{d+1} \in \Lambda_{d+1}} \psi_{(d,\star)}(x_d, \lambda_{d+1}) - V^*_{(d+1,\star,\mathbb{E})}(\lambda_{d+1})$$

Theorem

By weak duality and restriction, we get $V_{(d,\star,\mathbb{E})} \leq V_{(d,\star)} \leq V_d$

If $ri\Big(dom(\psi_{(d,\star)}(x_d,\cdot)) - dom(\mathbb{E}V_{d+1}(\cdot))\Big) \neq \emptyset$ and $\mathcal{P}$ is convex then we have $V_{(d,\star,\mathbb{E})} \leq V_{(d,\star)} = V_d$

There are efficient ways to compute the lower bounds $V_{(d,\star,\mathbb{E})}$

Numerical efficiency of deterministic weights decomposition

$$\overbrace{V_{(d,\star,\mathbb{E})}(x_d) = \sup_{\lambda_{d+1}\in\Lambda_{d+1}} \underbrace{\psi_{(d,\star)}(x_d,\lambda_{d+1})}_{\text{Hard to compute}} - V^*_{(d+1,\star,\mathbb{E})}(\lambda_{d+1})}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute $\psi_{(d,\star)}(x,\lambda)$ for each couple (x,λ) and each day d but

- Under Monotonicity Assumption,
  we can restrict to positive weights $\lambda \geq 0$
- We can exploit periodicity of the problem $\psi_{(d,\star)} = \psi_{(0,\star)}$
- We can parallelize $\psi_{(d,\star)}$ computation on day d

We will use the daily value functions upper and lower bounds

Back to daily intraday problems with final costs

We obtained two bounds $V_{(d,\star,\mathbb{E})} \leq V_d \leq V_{(d,\geq,\mathbb{X}_{d+1})}$

Now we can solve all intraday problems with a final cost

$$\min_{\boldsymbol{X}_{d+1},\boldsymbol{U}_d} \mathbb{E}\left[L_d(x,\boldsymbol{U}_d,\boldsymbol{W}_d) + \widetilde{V}_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(x,\boldsymbol{U}_d,\boldsymbol{W}_d)$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

with $\widetilde{V}_{d+1} = V_{(d,\geq,\mathbb{X}_{d+1})}$ or $\widetilde{V}_{d+1} = V_{(d,\star,\mathbb{E})}$

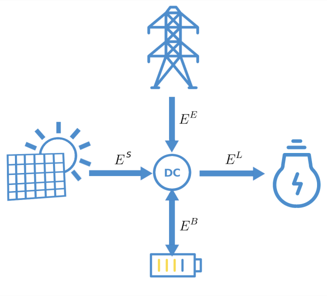
We obtain one targets and one weights minute scale policies

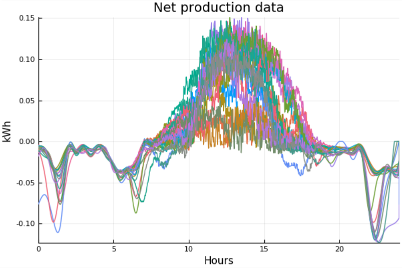
Numerical results

We present numerical results associated to two real use cases

Common data: load/production from a house with solar panels

1. Managing a given battery charge and health on 5 days
   to compare our algorithms to references on a “small” instance
2. Managing batteries purchases, charge and health on 7300 days
   to show that targets decomposition scales





Application 1: managing charge and aging of a battery

We control a battery

- capacity $c_0 = 13$ kWh
- $h_{0,0} = 100$ kWh of exchangeable energy (4 cycles remaining)
- over $D = 5$ days or $D \times M = 7200$ minutes
- with 1 day periodicity

We compare 4 algorithms

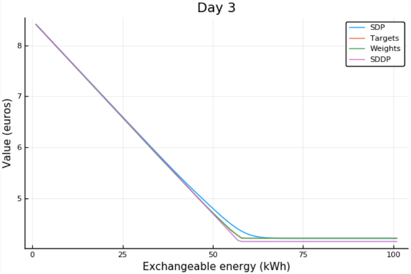
1. Stochastic dynamic programming
2. Stochastic dual dynamic programming
3. Targets decomposition (+ SDDP for intraday problems)
4. Weights decomposition (+ SDP for intraday problems)

Decomposition algorithms provide tighter bounds

We know that

- $V_d^{sddp} \leq V_d \leq V_d^{sdp}$
- $V_{(d,\star,\mathbb{E})} \leq V_d \leq V_{(d,\geq,\mathbb{X}_{d+1})}$

We observe that $V_d^{sddp} \leq V_{(d,\star,\mathbb{E})} \leq V_{(d,\geq,\mathbb{X}_{d+1})} \leq V_d^{sdp}$



We beat SDP and SDDP (that cannot fully handle 7200 stages)

Computation times and convergence

	SDP	**Weights**	**SDDP**	**Targets**
Total time (with parallelization)	22.5 min	5.0 min	3.6 min	0.41 min
Gap ($200 \times \frac{mc-v}{mc+v}$)	0.91 %	0.32 %	0.90 %	0.28 %

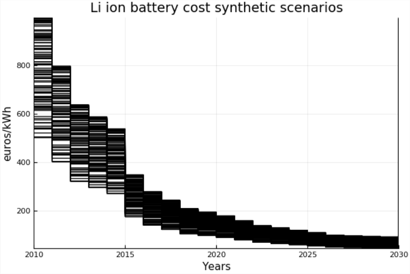
The Gap is between Monte Carlo simulation (upper bound)

and value functions at time 0

- Decomposition algorithms display smaller gaps
- Targets decompositon + SDDP is faster than SDDP
- Weights decomposition + SDP is faster than SDP

Application 2: managing batteries purchases, charge and aging

- 20 years, $10,512,000$ minutes, 1 day periodicity
- Battery capacity between 0 and 20 kWh
- Synthetic scenarios for batteries prices



SDP and SDDP fail to solve such a problem over $10,512,000$ stages!

Target decomposed SDDP solves $10,512,000$ stages problems

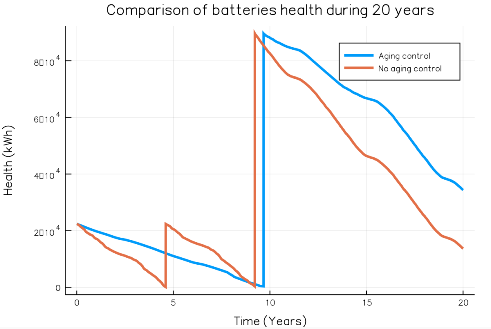
$$\overbrace{V_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\underbrace{\phi_{(d,\geq)}(x, X)}_{\text{Computing } \phi_{(d,\geq)}(\cdot,\cdot) \text{ with SDDP takes 60 min}} + V_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)}^{\text{Computing daily value functions by dynamic programming takes 45 min}}$$

Complexity: 45 min $+$ $D \times 60$ min

- Periodicity: 45 min $+$ $N \times 60$ min with $N << D$
- Parallelization: 45 min $+$ 60 min

Does it pay to control aging?

We draw one battery prices scenario and one solar/demand scenario over $10,512,000$ minutes and simulate the policy of targets algorithm



We make a simulation of $10,512,000$ decisions in 45 minutes

We compare to a policy that does not control aging

- Without aging control: 3 battery purchases
- With aging control: 2 battery purchases

It pays to control aging with targets decomposition!

Conclusion

1. **We have solved problems with millions of time steps using targets decomposed SDDP**
2. **We have designed control strategies for sizing/charging/aging/investment of batteries**
3. **We have used our algorithms to improve results obtained with algorithms sensitive to the number of time steps (SDP, SDDP)**

Merci for your attention