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A stochastic multi-item lot-sizing problem with bounded number of setups

Séminaire des doctorants

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Outline

- Business problem and model
- Deterministic model
- Stochastic model
- Numerical experiments

Business problem

- Context: production function of the Supply Chain for one assembly line
- Objective: reduction of holding costs
- Main constraints: industrial flexibility and “high service level”
- Typical horizon: 10 to 15 weeks
- Typical time step: 1 week
- Input data:
  - a set of references $r \in \mathcal{R}$
  - capacity of the line
  - demand for each reference and each week

Classical problem: Capacitated Lot-Sizing Problem (CLSP)

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} \sum_{r \in \mathcal{R}} (h_t^r s_t^r + c_t^r x_t^r) & & \\
\text{s.t.} \quad & s_t^r = s_{t-1}^r + q_t^r - d_t^r & \forall t, \forall r & \\
& \sum_{r \in \mathcal{R}} q_t^r \leq 1 & \forall t & \qquad \text{(CLSP)} \\
& q_t^r \leq x_t^r & \forall t, \forall r & \\
& x_t^r \in \{0, 1\} & \forall t, \forall r & \\
& q_t^r,\ s_t^r \geq 0 & \forall t, \forall r &
\end{aligned}
$$

- with ($r \in \mathcal{R}$ for references, $t \in [T]$ for weeks):

input data		variables	
h_t^r	holding cost	s_t^r	inventory level
c_t^r	setup cost	q_t^r	produced quantity
d_t^r	demand	x_t^r	setup variable

Disadvantage of CLSP formulation (according to Argon Consulting)

- Hard to compare holding costs and setup costs
- Number of setups is a "technical" constraint
  - Given by operational level
  - Represent scheduling constraints (which are neglected at tactical level)

Multi-item lot-sizing problem with bounded number of setups

$$
\begin{array}{lll}
\min & \displaystyle\sum_{t=1}^{T}\sum_{r\in\mathcal{R}} (h_t^r s_t^r + \cancel{c_t^r x_t^r}) & \\
\text{s.t.} & s_t^r = s_{t-1}^r + q_t^r - d_t^r & \forall t, \forall r \\
& \displaystyle\sum_{r\in\mathcal{R}} q_t^r \leq 1 & \forall t \\
& q_t^r \leq x_t^r & \forall t, \forall r \\
& \displaystyle\sum_{r\in\mathcal{R}} x_t^r \leq N & \forall t \\
& x_t^r \in \{0,1\} & \forall t, \forall r \\
& q_t^r,\ s_t^r \geq 0 & \forall t, \forall r
\end{array}
\qquad (P)
$$

- Bounded number of setups per week (N)
- Easier for industrials to quantify holding costs and N

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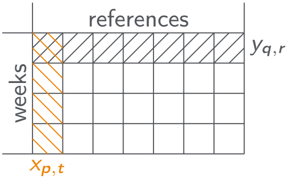
The deterministic model is hard

- (P) is $\mathcal{NP}$-hard (reducing 3-PARTITION)
  - Decide if there is a solution when $N = 1$ is polynomial
  - Cases $N = 1$ and $N = 2$ still open
- Continuous relaxation of (P) does NOT depend on N
- 2 natural extended formulations:
  - 1 binary variable $x_{p,t}$ where $p \in \binom{\mathcal{R}}{N}$ per possible plan for a week
  - 1 binary variable $y_{q,r}$ where $q \in 2^{[T]}$ per possible plan for a reference



  - Same continuous relaxations than compact formulation

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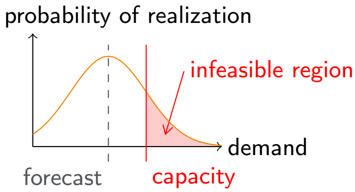
Need for backlog

- Mathematical reason:
  - In general, there is no feasible solution
  - Simple example:
    - bounded capacity C
    - demand = Gaussian noise around forecast



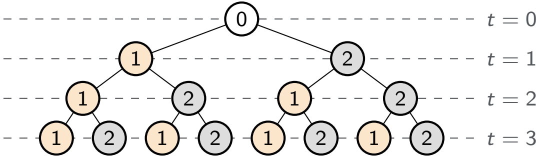
- Industrial reason:
  - Negative inventories are commercial constraints
    $\Longrightarrow$ "soft" constraints
  - Firms can deliver late

$$
\begin{aligned}
\min \quad & \mathbb{E}\left[\sum_{t=1}^{T}\sum_{r\in\mathcal{R}}\left(h_t^r \tilde{s}_t^r + \gamma \boldsymbol{b}_t^r\right)\right] & & \\
\text{s.t.} \quad & \boldsymbol{s}_t^r = \tilde{\boldsymbol{s}}_t^r - \boldsymbol{b}_t^r & & \forall t, \forall r \\
& \boldsymbol{s}_t^r = \boldsymbol{s}_{t-1}^r + \boldsymbol{q}_t^r - \boldsymbol{d}_t^r & & \forall t, \forall r \\
& \sum_{r\in\mathcal{R}} \boldsymbol{q}_t^r \leq 1 & & \forall t \\
& \boldsymbol{q}_t^r \leq \boldsymbol{x}_t^r & & \forall t, \forall r \\
& \sum_{r\in\mathcal{R}} \boldsymbol{x}_t^r \leq N & & \forall t \\
& \boldsymbol{x}_t^r \in \{0,1\} & & \forall t, \forall r \\
& \boldsymbol{q}_t^r,\ \tilde{\boldsymbol{s}}_t^r,\ \boldsymbol{b}_t^r \geq 0 & & \forall t, \forall r \\
& \sigma\left(\boldsymbol{q}_t^r\right),\ \sigma\left(\boldsymbol{x}_t^r\right) \subset \sigma\left(\left(\boldsymbol{d}_0^r, \dots, \boldsymbol{d}_t^r\right)_{r\in\mathcal{R}}\right) & & \forall t, \forall r
\end{aligned}
\qquad (S)
$$

with: γ backlog penalization, $\tilde{\boldsymbol{s}}_t^r$ inventory level,
$\boldsymbol{b}_t^r$ backlog quantity

Stochastic model: size difficulty

- Extensive formulation leads to a huge number of variables

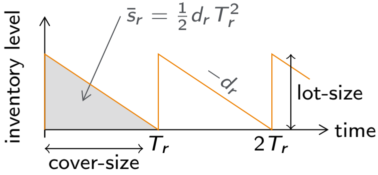


  - example: for each references, 2 independent possibilities for demand
    $\Longrightarrow$ number of variables multiplied by $\left(2^T\right)^{|\mathcal{R}|}$
  - for $T = 10$, $|\mathcal{R}| = 10$, numbers of variables $\approx 10^{30}$
- Need for heuristics to solve (S)
  - lot-size and cover-size
  - open-loop feedback approach
  - repeated two-stage stochastic programming approach

Aside: computing lot-size and cover-size

- Simplified model:
  - Constant demand for each reference over time
  - Aggregated demand

Inventory level of reference r:



Corresponding program to solve:

$$\begin{aligned} \min \quad & \sum_{r \in \mathcal{R}} \frac{1}{2} h_r d_r T_r \\ \text{s.t.} \quad & \sum_{r \in \mathcal{R}} \frac{1}{T_r} = N \\ & T_r > 0 \qquad \forall r \end{aligned}$$

- Closed-form expressions of solutions

$$\nu_r^* = \frac{1}{T_r^*} = \frac{N\sqrt{h_r d_r}}{\sum_{p \in \mathcal{R}} \sqrt{h_p d_p}} \quad \text{and} \quad \text{Cost} = \frac{1}{2N} \left(\sum_{r \in \mathcal{R}} \sqrt{h_r d_r} \right)^2$$

- Closed-form expressions of solutions for stochastic case

Strategy: lot-size and cover-size

- Heuristic parameters: safety stock for each reference

foreach $r \in \mathcal{R}$ **do**
  Compute cover-size T_r / lot size $\ell_r = d_r T_r$;
for *week* **from** *1* **to** *T* **do**
  foreach $r \in \mathcal{R}$ **do**
    Observe inventory level of r;
    if *current inventory level* < *safety stock* **then**
      case *lot-size:* produce quantity ℓ_r;
      case *cover-size:* produce the cumulated expected demand for the T_r next weeks;

- Example:
  - For a reference r, $T_r = 2$ weeks
  - Expected demand is:

week t	1	2	3	4	5
expected demand f_t	2	3	5	4	1

  - If current inventory level < safety stock at week 1, we must produce:

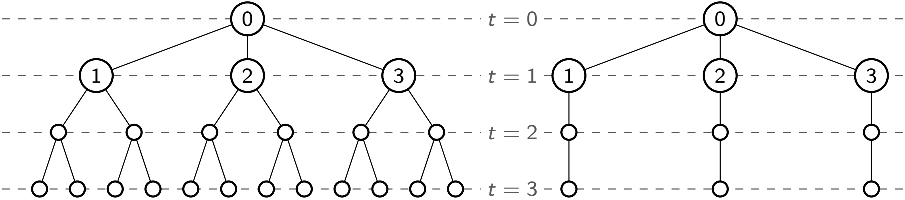
$$f_1 + f_2 = 2 + 3 = 5 \text{ units of reference } r$$

Strategy: open-loop feedback approach

- At week t:
  - Observe current inventory level
  - Solve deterministic version of (S) where the random variable $\boldsymbol{d}_t^r$ is replaced by the deterministic expected demand
    - It is a Mixed Integer Program
    - Almost program (P) but with backlog
  - Set production decisions for week t

Strategy: repeated two-stage stochastic programming approach

- At week t:
  - Observe current inventory level
  - Construct a fan of demand scenarios to approximate the tree of scenarios in (S)



Complete tree of scenarios

Fan of scenarios

  - Solve (S)
  - Set production decisions for week t

There is a lot of possible forecasts

- Static deterministic forecast
  - Expectation, median...
- Adaptative deterministic forecast
  - Autoregressive process, time series...
- Stochastic forecast
  - Tree of scenario, fan of scenarios...

Every strategy works even if we do not know distribution laws.
We just need a forecast function!

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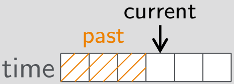
Demand realization

Strategy → Forecast → Data →

Simulator

- Start at week $t = 0$
- For each week t:



  - Observe inventory level
  - Use strategy and forecast to compute the production planning
  - Set production for current week
- Return KPI

→ Key performance indicators:

- Holding costs
- Backlog costs
- Cycle service level
- Fill rate service level
- . . .

Data of the instances

- We use historical data from industrial
- Numerical values:
  - Horizon $T = 13$ weeks
  - $|\mathcal{R}| = 30$ references
  - Demands $0 \leq d_t^r \leq 4000$ units
  - Weekly capacity $C \approx 13000$ units
  - Weekly number of setups $N = 10$
  - Holding costs $50 \leq h_t^r \leq 80$ per units

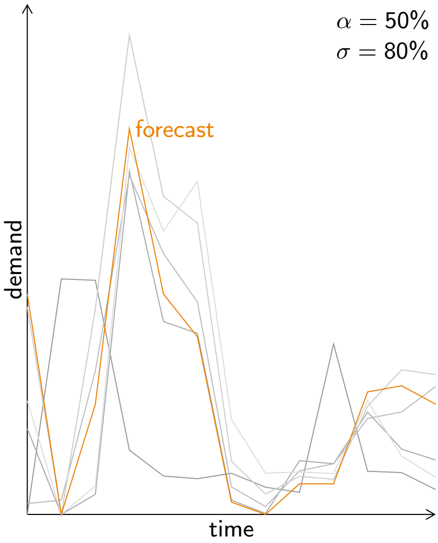
- Autoregressive Process (AR1).
  For each reference r,

$$d_{t+1} = f_{t+1} + \underbrace{\alpha e_t + (1-\alpha)\,\epsilon_{t+1}}_{e_{t+1}}$$

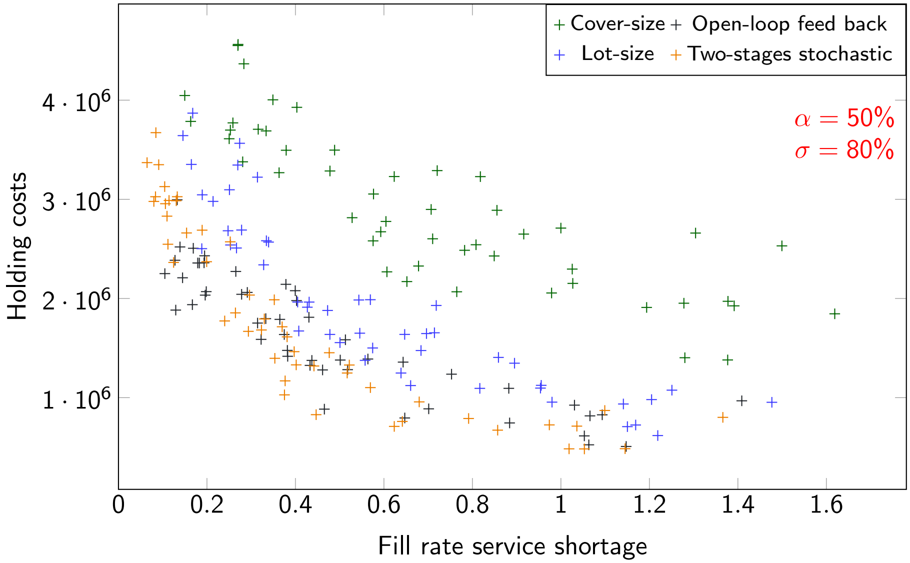
where (at week t):

  - d_t is the demand
  - f_t is the forecast
  - e_t is the forecast error
  - $\epsilon_t \sim \mathcal{N}(0, \sigma f_t)$ is a white noise

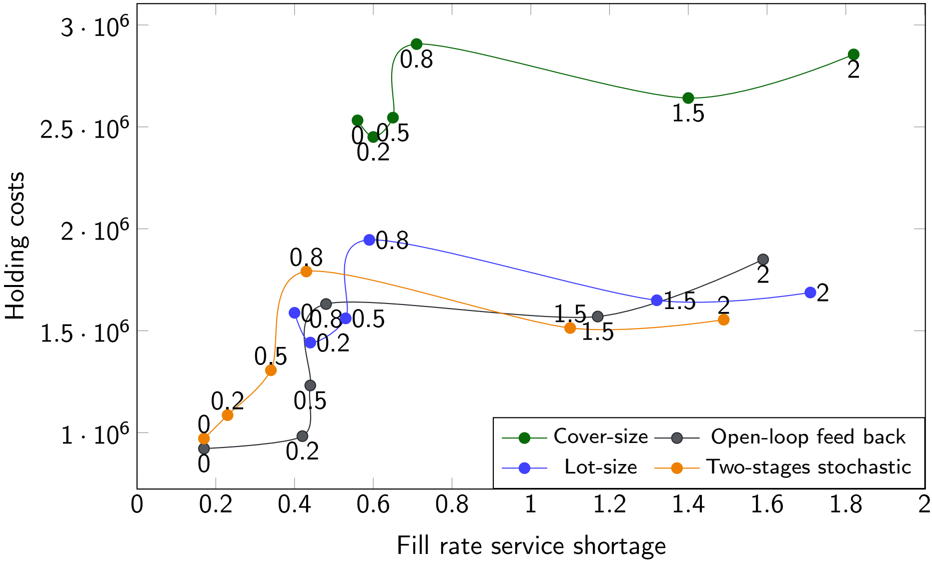
- 2 parameters:
  - $\alpha \in [0,1]$ proportion error/noise
  - σ is the volatility.



Results: holding costs for several realizations of demand



Results: holding costs for several values of volatility



Thanks for your attention!

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