Resource constrained shortest path algorithm for EDF short-term thermal production planning problem

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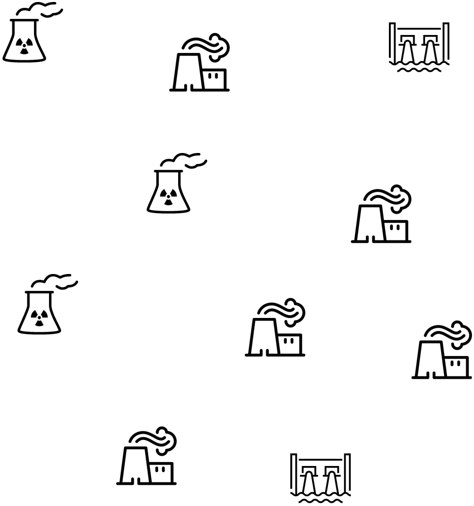
EDF - Global Problem



Generation units

- ~ 60 nuclear
- ~ 100 thermal
- ~ 500 hydraulic

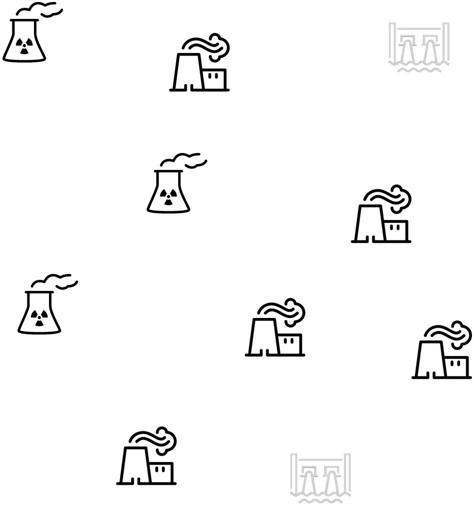
Technically feasible production schedules

Min operating cost

Horizon: $24\text{h} + 24\text{h}$

Time limit: 15m

EDF - Global Problem



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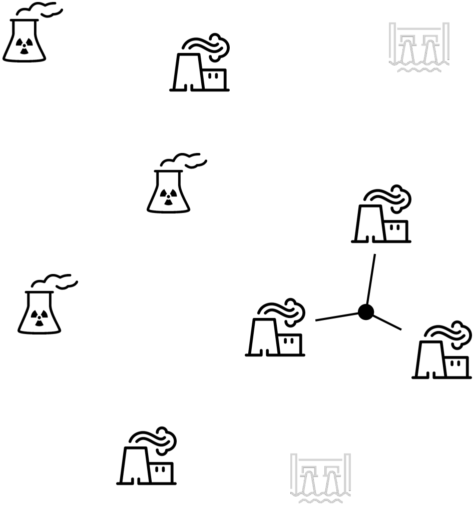

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Production Plan - Local Problem

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Production Plan - Local Problem

startup · #startups

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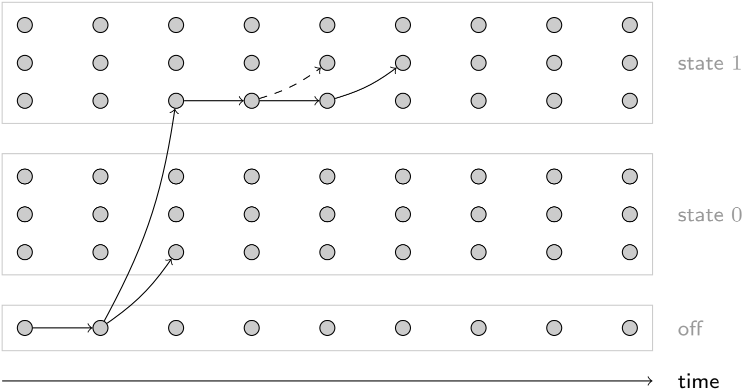
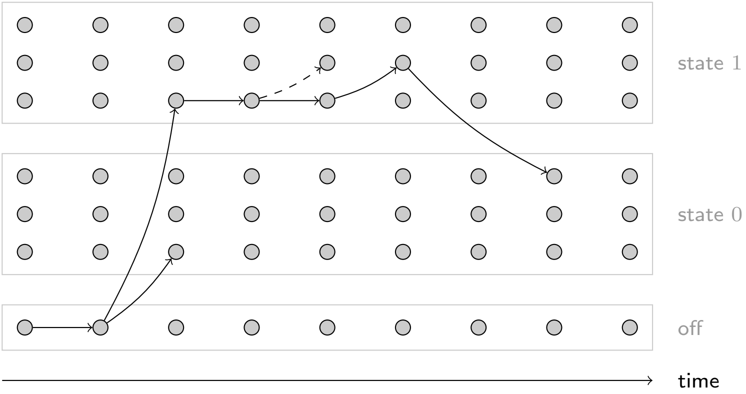


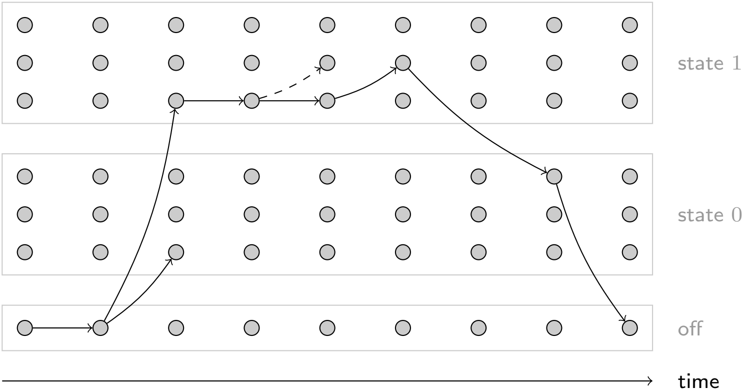
startup · #startups · min_duration_production_level

Production Plan - Local Problem



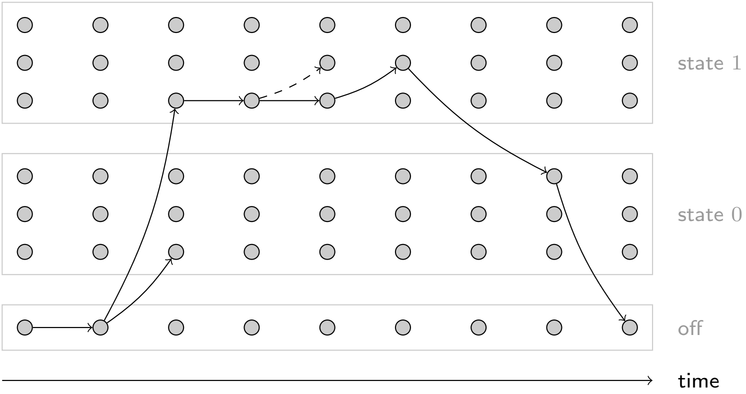
startup · #startups · min_duration_production_level
min_duration_power_state · #modulations

Production Plan - Local Problem



startup · #startups · min_duration_production_level
min_duration_power_state · #modulations · shutdown

Production Plan - Local Problem



startup · #startups · min_duration_production_level
min_duration_power_state · #modulations · shutdown
min/max_power_state · min/max_increase/decrease
#deep_decreases

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The Unit Commitment Problem (1)

Assume we can generate all technically feasible production plans $\mathcal{P}_i$ for each plant $i \in V$.

$$
\begin{array}{lrcll}
\text{maximize} & \displaystyle\sum_{p \in \mathcal{P}} c_p x_p & & & \\
\text{subject to} & \displaystyle\sum_{p \in \mathcal{P}_i} x_p & = & 1 & \forall i \in V \\
 & x_p & \in & \{0,1\} & \forall p \in \mathcal{P}
\end{array}
$$

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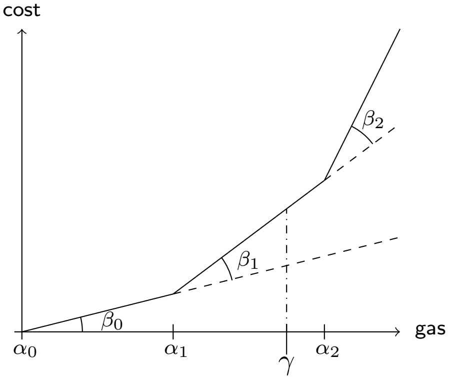
What makes the problem more difficult?

- Linking constraints: Shared gas stock
- Piecewise linear cost function on gas consumption γ

Excursion: Piecewise Linear Objective Function



$$\begin{aligned} \min \quad & \sum_k \beta_k \xi_k \\ \text{s.t.} \quad & \gamma - \alpha_k \leq \xi_k \quad \forall k \\ & \xi_k \geq 0 \quad \forall k \end{aligned}$$

The Unit Commitment Problem (2)

Let j describe a group of plants.

$$
\begin{array}{lrcll}
\text{maximize} & \displaystyle\sum_{p\in\mathcal{P}} c_p x_p - \sum_j \sum_k \beta_{jk}\xi_{jk} & & & \\
\text{subject to} & \displaystyle\sum_{p\in\mathcal{P}_i} x_p & = & 1 & \forall i \in V \\
 & \displaystyle\sum_{p\in\mathcal{P}_j} \gamma_p x_p - \alpha_{jk} & \leq & \xi_{jk} & \forall j,k \\
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& \sum_{p \in \mathcal{P}_j} \gamma_p x_p & \geq & g_j & \forall j \\
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& x_p & \in & \{0,1\} & \forall p \in \mathcal{P} \\
& \xi_{jk} & \geq & 0 & \forall j,k
\end{array}
$$

Why not solving it directly?

Column Generation in General

1. Start with subset of variables $\widetilde{\mathcal{P}} \subset \mathcal{P}$
2. Solve relaxed version of primal problem $x_p \geq 0$
3. Get dual variables for each constraint λ, μ, π
4. Find violated constraint in the dual problem
5. Add variable x_p to the problem $\widetilde{\mathcal{P}} \cup \{p\}$

Applied Column Generation

Start with an inital set of feasible paths $\widetilde{\mathcal{P}} \subset P$.

$$
\begin{array}{llcll}
\text{maximize} & \displaystyle\sum_{p\in\widetilde{\mathcal{P}}} c_p x_p - \sum_j \sum_k \beta_{jk}\xi_{jk} & & & \\
\text{subject to} & \displaystyle\sum_{p\in\widetilde{\mathcal{P}}_i} x_p & = & 1 & \forall i \in V \\
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Applied Column Generation

Start with an inital set of feasible paths $\widetilde{\mathcal{P}} \subset P$.

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\begin{array}{llcll|l}
\text{maximize} & \sum_{p \in \widetilde{\mathcal{P}}} c_p x_p - \sum_j \sum_k \beta_{jk} \xi_{jk} & & & & \\
\text{subject to} & \sum_{p \in \widetilde{\mathcal{P}}_i} x_p & = & 1 & \forall i \in V & \lambda_i \\
 & \sum_{p \in \widetilde{\mathcal{P}}_j} \gamma_p x_p & \geq & g_j & \forall j & \mu_j^0 \\
 & \sum_{p \in \widetilde{\mathcal{P}}_j} \gamma_p x_p & \leq & G_j & \forall j & \mu_j^1 \\
 & \sum_{p \in \widetilde{\mathcal{P}}_j} \gamma_p x_p - \alpha_{jk} & \leq & \xi_{jk} & \forall j, k & \pi_{jk} \\
 & x_p & \geq & 0 & \forall p \in \widetilde{\mathcal{P}} & \\
 & \xi_{jk} & \geq & 0 & \forall j, k &
\end{array}
$$

Dual and Pricing Problem

Dual

$$
\begin{aligned}
\min \quad & -\sum_{i\in V}\lambda_i + \sum_{j\in J} -\mu_j^0 g_j + \mu_j^1 G_j + \sum_k \pi_{jk}\alpha_{jk} \\
\text{s.t.} \quad & -\beta_{jk} + \pi_{jk} \leq 0 \quad \forall j,k \\
& c_p + \lambda_{i(p)} + \gamma_p(\mu^0_{j(p)} - \mu^1_{j(p)} - \sum_k \pi_{j(p)k}) \leq 0 \quad \forall p \in \widetilde{\mathcal{P}}
\end{aligned}
$$

Dual and Pricing Problem

Dual

$$\begin{aligned}
\min \quad & -\sum_{i\in V}\lambda_i + \sum_{j\in J} -\mu_j^0 g_j + \mu_j^1 G_j + \sum_k \pi_{jk}\alpha_{jk} \\
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\end{aligned}$$

Pricing

$$\max_{p\in P} \quad c_p + \lambda_{i(p)} + \gamma_p(\mu^0_{j(p)} - \mu^1_{j(p)} - \sum_k \pi_{j(p)k})$$

Dual and Pricing Problem

Dual

$$\begin{array}{lrcll} \min & -\sum_{i \in V} \lambda_i + \sum_{j \in J} -\mu_j^0 g_j + \mu_j^1 G_j + \sum_k \pi_{jk} \alpha_{jk} & & & \\ \text{s.t.} & -\beta_{jk} + \pi_{jk} & \leq & 0 & \forall j, k \\ & c_p + \lambda_{i(p)} + \gamma_p(\mu_{j(p)}^0 - \mu_{j(p)}^1 - \sum_k \pi_{j(p)k}) & \leq & 0 & \forall p \in \widetilde{\mathcal{P}} \end{array}$$

Pricing

$$\max_{p \in P} \quad c_p + \lambda_{i(p)} + \gamma_p(\mu_{j(p)}^0 - \mu_{j(p)}^1 - \sum_k \pi_{j(p)k})$$

Main Task

Solve pricing problem fast!

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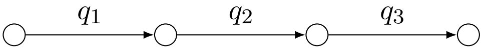
Shortest Path in an Ordered Monoid

For each arc a a resource $q_a \in \mathcal{R}$

- Associative binary operator $\oplus$: path resources
- Neutral element 0: empty path



$$q_P = q_1 \oplus q_2 \oplus q_3$$

$(\mathcal{R}, \oplus)$ is a monoid.

- An order $\preceq$ compatible with $\oplus$: $$q \preceq \tilde{q} \Rightarrow \begin{cases} r \oplus q \preceq r \oplus \tilde{q} \\ q \oplus r \preceq \tilde{q} \oplus r \end{cases}$$

$(\mathcal{R}, \oplus, \preceq)$ is an ordered monoid.

- Non-decreasing cost c and constraint ρ functions.

Shortest Path with Resources in an Ordered Monoid

Given an ordered monoid $(\mathcal{R}, \oplus, \preceq)$

Input:

- Digraph $D = (V, A)$
- Two vertices $o, d \in V$
- Resources $q_a \in \mathcal{R}$
- Two non-decreasing oracles $c : \mathcal{R} \to \mathbb{R}$ $\rho : \mathcal{R} \to \{0, 1\}$

Output:

- An o-d path P such that

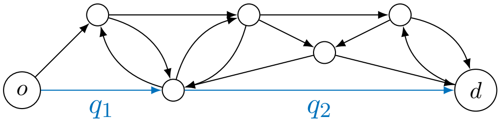
$$\rho\left(\bigoplus_{a \in P} q_a\right) = 0$$

which minimizes

$$c\left(\bigoplus_{a \in P} q_a\right)$$

Cost and constraint(s):

- non-linear(s)



$q_P = q_1 \oplus q_2$ $\quad\rho(q_P) = 0$

Example 1: Usual Resource Constrained Shortest Path

Input:

- Digraph $D = (V, A)$
- Origin o, Destination d
- Costs $c_a \in \mathbb{R}$
- Weights $w_a^i \in \mathbb{R}$ for $i \in [n]$
- Thresholds $W^i \in \mathbb{R}$ for $i \in [n]$

Output:

- An o-d path P such that

$$\sum_{a \in P} w_a^i \leq W^i \quad \forall i \in [n]$$

which minimizes $\sum_{a \in P} c_a$

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Model:

- $\mathcal{R} = \mathbb{R}^{n+1}$
- $q_a = (c_a, w_a^1, \ldots, w_a^n)$
- $c : ((q^0, \ldots, q^n)) \mapsto q^0$
- $\rho : ((q^0, \ldots, q^n)) \mapsto \max_i \mathbb{1}_{q^i > W^i}$

Example 2: Restricting Startups

Input:

- Digraph $D = (V, A)$
- Origin o, destination d
- $w_a = \begin{cases} 1, & \text{if startup arc,} \\ 0, & \text{otherwise.} \end{cases}$
- Max startups W^{start}

Output:

- An o-d path P of minimum cost such that the number of startups per plant is not greater than W^{start}.

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- Digraph $D = (V, A)$
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- Max startups W^{start}

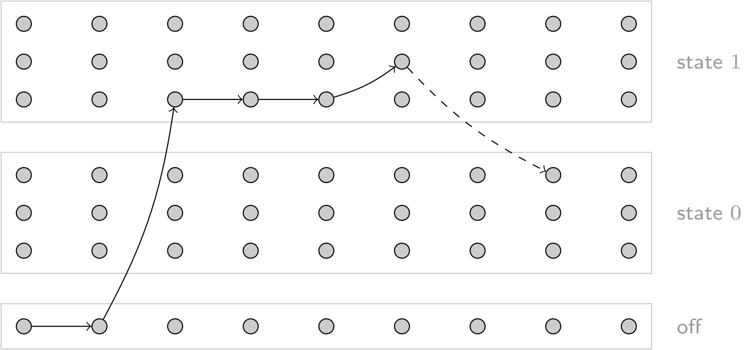
Output:

- An o-d path P of minimum cost such that the number of startups per plant is not greater than W^{start}.

Model:

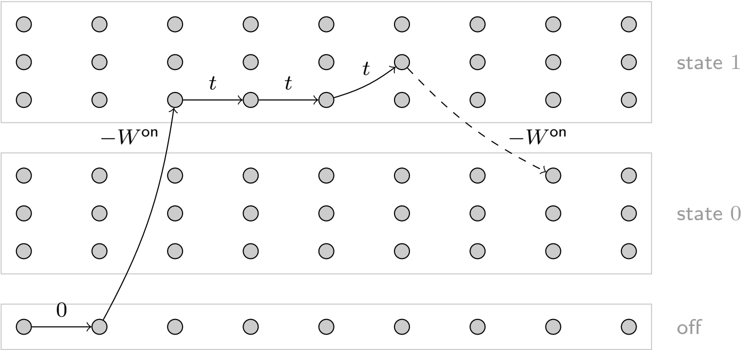
- $\mathcal{R} = \mathbb{R}^2$
- $q_a = (c_a, w_a)$
- $c : ((q^0, q^1)) \mapsto q^0$
- $\rho : ((q^0, q^1) \mapsto \mathbb{1}_{q^1 > W^{\text{start}}}$

Example 3: Minumum Duration in Online State



Stay in online state for at least W^{on}.

Example 3: Minumum Duration in Online State



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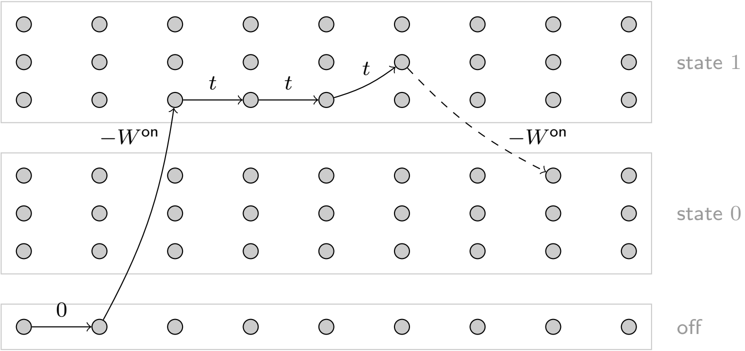

Stay in online state for at least W^{on}.

$$(c_a, w_a^1) \oplus (c_{a'}, w_{a'}^1) = \begin{cases} \infty & \text{, if } w_a^1 < 0 \wedge w_{a'}^1 < 0, \\ w_{a'}^1 & \text{, if } w_a^1 \geq 0 \wedge w_{a'}^1 < 0, \\ w_a^1 + w_{a'}^1 & \text{, otherwise} \end{cases}$$

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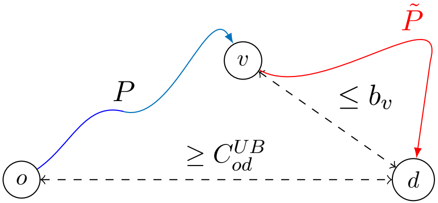
Usual A* algorithm



- $q_P \in \mathbb{R}$
- $C_{od}^{UB} \geq \min_{P \in \mathcal{P}_{o,d}} q_P$
- $b_v \leq q_P, \forall P \in \mathcal{P}_{vd}$

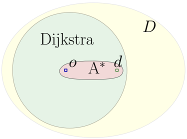
A path $P \in \mathcal{P}_{ov}$ satisfying $q_P + b_v > C_{od}^{UB}$ is not the subpath of an optimal path.

A* algorithm: a Branch & Bound

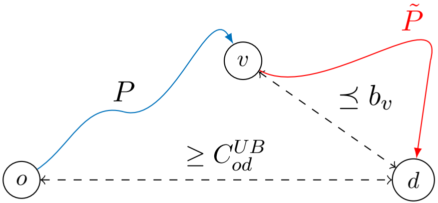
- Generate all the paths satisfying

$$q_P + b_v \leq C_{od}^{UB}$$

- Update C_{od}^{UB}



Generalized A* algorithm



- $q_P \in \mathcal{R}$
- $C_{od}^{UB} \geq \min_{P|\rho(P)=0} c(q_P)$
- $b_v \preceq q_{\tilde{P}}, \forall \tilde{P} \in \mathcal{P}_{vd}$

A path $P \in \mathcal{P}_{ov}$ satisfying $c(q_P \oplus b_v) > C_{od}^{UB}$ or $\rho(q_P \oplus b_v) = 1$ is not the subpath of an optimal path.

Generalized A* Algorithm: a Branch & Bound

- Generate all the paths satisfying

$$c(q_P \oplus b_v) \leq C_{od}^{UB} \quad \text{and} \quad \rho(q_P \oplus b_v) = 0 \qquad \text{(Low)}$$

- Update C_{od}^{UB}

Generic enumeration algorithm

Preprocessing.

$L \leftarrow$ empty path in o

$c_{od}^{UB} \leftarrow +\infty$.

While L is not empty:

- Extract from L a path P of minimum key.
- If $v = d$ and $\rho(P) = 0$, $c_{od}^{UB} \leftarrow \min(c_{od}^{UB}, c(P))$.
- Test if P must be extended. If yes:
  - for each $a \in \delta^+(v)$, add $P + a$ to L.

L: cand. paths list.
c_{od}^{UB}: Upper bound on optimal path cost.
v: destination of P.

Add. structures

b_v: lower bound on v-d paths q_P
M_v: list of non dominated o-v paths

Algorithm	Test	Key
Generalized A*	(Low)	$c(q_P \oplus b_v)$
Label dominance	(Dom)	$c(q_P)$
Label correcting	(Dom), (Low)	$c(q_P \oplus b_v)$

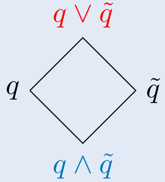
Bound Computation

Definition: *lattice*

A partially ordered set $(\mathcal{R}, \preceq)$ is a lattice if any pair $(q, \tilde{q})$ admits:

A greatest lower bound or *meet* denoted $q \wedge \tilde{q}$

$$\left.\begin{array}{l} b \preceq q \\ b \preceq \tilde{q} \end{array}\right\} \Leftrightarrow b \preceq q \wedge \tilde{q}$$



A least upper bound or *join* denoted $q \vee \tilde{q}$

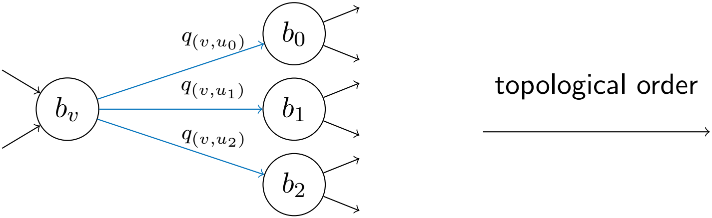
$$\left.\begin{array}{l} b \succeq q \\ b \succeq \tilde{q} \end{array}\right\} \Leftrightarrow b \succeq q \vee \tilde{q}$$

Example:

$(\mathbb{R}^2, \leq)$ endowed $\leq$ with the product order

- $q \wedge \tilde{q} = (\min(q_1, \tilde{q}_1), \min(q_2, \tilde{q}_2))$
- $q \vee \tilde{q} = (\max(q_1, \tilde{q}_1), \max(q_2, \tilde{q}_2))$

Generalized Ford-Bellman algorithm



If there is no cycles of negative cost, b_v can be computed by the generalized dynamic programming equation:

$$b_v = \begin{cases} 0 & \text{,if } v = d \\ \bigwedge \left(b_v, \bigwedge_{u \in N^+(v)} \left(q_{(v,u)} \oplus b_u \right) \right) & \text{,if } v \neq d \end{cases}$$

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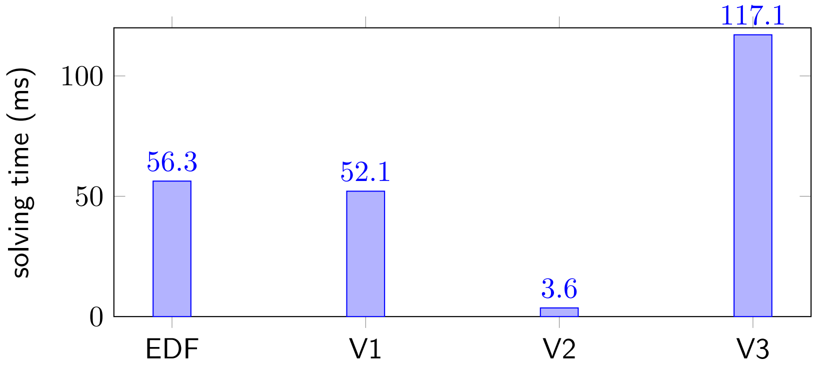
Some Numbers

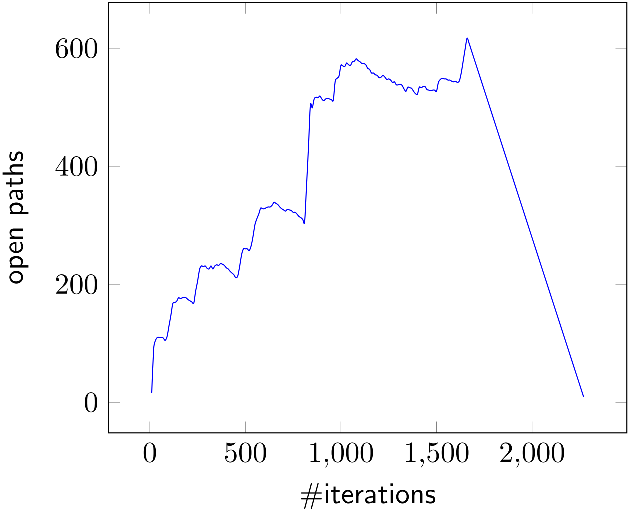
- Plants
  - 8 gas plants $(3, 3, 2)$
  - 97 non-gas plants
- Size of graph (depends on the model)
  - ~ 2.000 nodes
  - ~ 10.000 arcs

Solving Non-Gas Subproblem



	EDF	V1	V2	V3
selection	node	node	path	path
key	early date	early date	$q_p \oplus b_v$	q_p
#iter	2.100	1.914	3.596	230.763
#dis dom	190k	137k	9k	201k
#dis bound	0	79k	9k	1.029k
#od paths	327	14	1	899
speedup	1.00x	0.93x	0.06x	2.08x

An Attempt to Explain



Solving Gas Subproblem

Pricing

$$\max_{p \in P} \quad c_p + \lambda_{i(p)} + \gamma_p(\mu^0_{j(p)} - \mu^1_{j(p)} - \sum_k \pi_{j(p)k})$$

dual	V2 (ms)	EDF (ms)
0	0.15068	44.0543
-1	1.26815	43.0639
-5	5.45315	52.6267
-10	7.33129	40.8962
-50	1.05291	28.4392

Column Generation of Gas Plants

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What was the Problem?

Assume we can generate all technically feasible production plans $\mathcal{P}_i$ for each plant $i \in V$.

$$\begin{aligned} \text{maximize} \quad & \sum_{p \in \mathcal{P}} c_p x_p - \sum_j \sum_k \beta_{jk} \xi_{jk} \\ \text{subject to} \quad & \qquad\qquad\qquad\qquad \dots \end{aligned}$$

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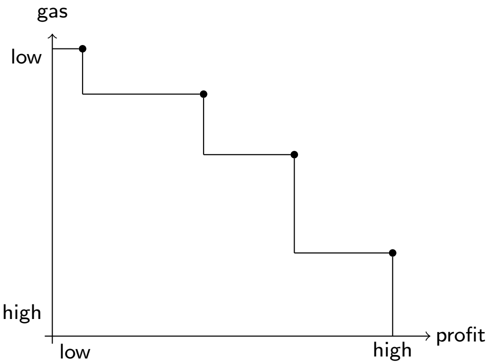
Try to compute all non-dominated paths based on profit and gas consumption.

Pareto Frontier



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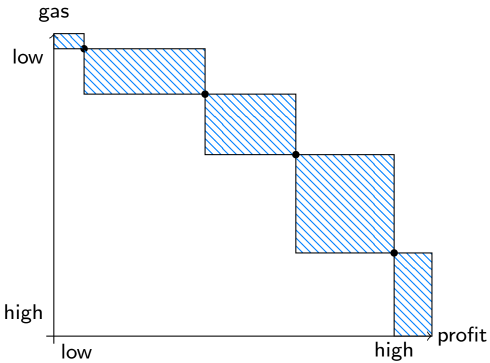

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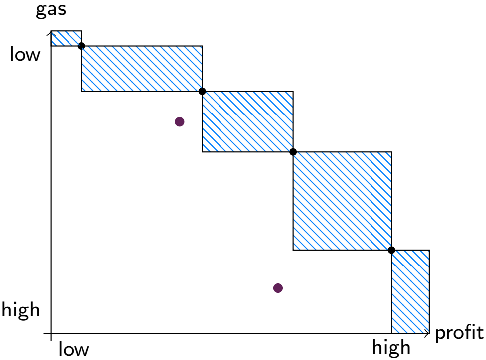
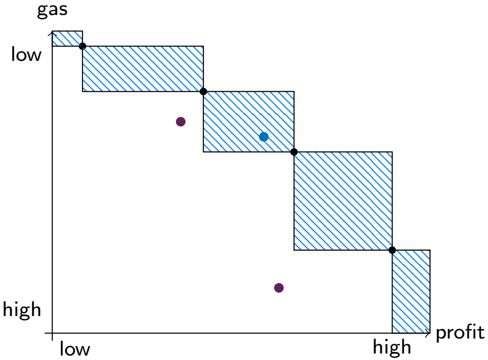


dominated by pareto

Pareto Frontier



dominated by pareto

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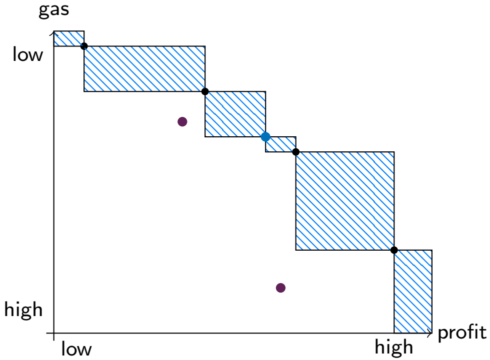
Pareto Frontier



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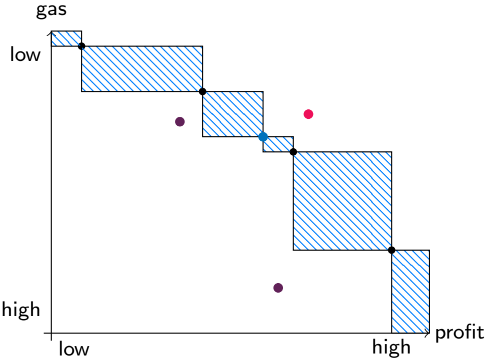

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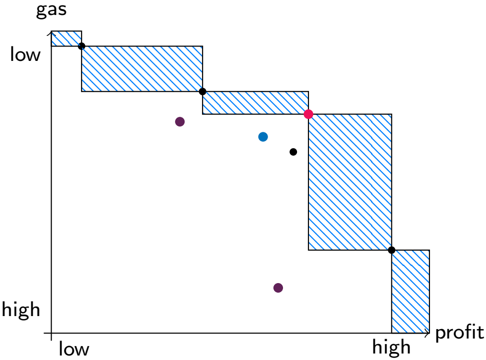

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How to Compute? RCSPP

Talking about failures!

Bound test before:

$$\text{discard if: } c(q_P \oplus b_v) \leq C_{od}^{LB} \qquad \text{(Low)}$$

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Bound test after:

for (*profit*, *gas*) in pareto:
    if $c_{\textit{profit}}(q_P \oplus b_v) \leq$ *profit* and $c_{\textit{gas}}(q_P \oplus b_v) \geq$ *gas*
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No solution after 8h...

How to Compute? Linear Combination

Still talking about failures!

$$c : ((\dots, \mathit{profit}, \dots, \mathit{gas}, \dots)) \mapsto \alpha \cdot \mathit{profit} + (1 - \alpha) \cdot \mathit{gas}$$

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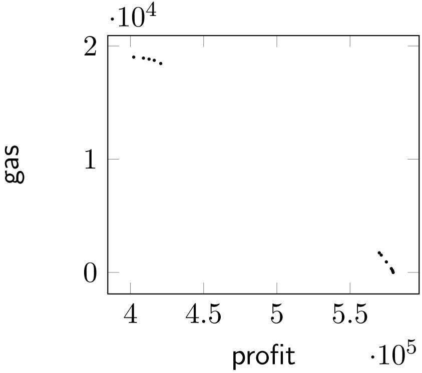

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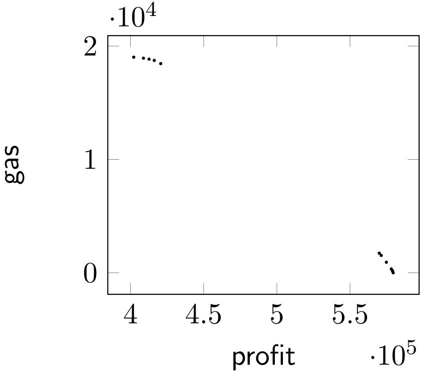


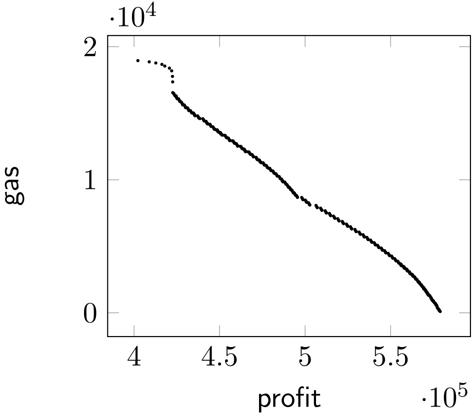
parteo front is non convex!

How to Compute? Upper Bound on Gas Consumption

$gasUpperBound \leftarrow INF$
while $gasUpperBound \geq 0$

- solve RCSPP($gasUpperBound$)
- add solution ($profit$, gas) to pareto
- $gasUpperBound \leftarrow gas - \epsilon$

How to Compute? Upper Bound on Gas Consumpition



Solving time: ~ 38s
Size pareto: ~ 600

Summary

- Redesign of the graph
- Modeling of RCSPP in lattice ordered monoid
- (Conditional) bound computation
- Solving of non-/gas pricing problems
- Column generation - Multi Unit Commitment Problem
- Pareto Frontier