Energy derivatives – Optimisation & Pricing

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Arnaud de Latour, arnaud.de-latour@edf.fr

Outline

- Energy derivatives modelling
  - A typical energy portfolio
  - Market modelling of some energy derivatives:
    Swaps, power plants, storage assets, load-curve contracts, ...

- Some pricing methods will be detailed for...
  - Spread options
  - Swing options

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ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- Gas storage modelling
- Swing options pricing
- Load curve contract modelling
Energy derivatives

Risk context and objectives

- Energy/electric utilities are exposed to a whole set of risk factors:
  - **Energy market risk**: electricity prices, fuel prices (coal, crude, gas, ...), emission prices, for both forward and spot prices
  - **Financial risk**: currency, interest rates, counterpart risk
  - **Volume risk**: industrial/technical risk (outages, inflows), physical risk (network), commercial risk (consumption and market shares), weather risk (temperature, cloud cover, ...)
  - **Regulatory risk**: politics and regulation, economic risk
  - **Climatic risk**, natural disasters

- The challenges in this context are the following:
  - Modelling **realistic prices** and their dependencies
  - **Pricing** forwards, derivatives components of the portfolio
  - Assessing risks and **hedging** structured products (computing Greeks) and more globally the whole portfolio
  - Designing models for a **risk management system** for a utility exposed to those risk factors...
Energy derivatives
A typical energy portfolio

Like any other commodity market, energy markets have their options on quoted futures. But, the most challenging problems come from the pricing, hedging and structuring of exotic tradable products linked to physical assets: can be called real derivatives.

Physical assets are modelled (and valuate) by using the real option theory.

- When traditional deterministic methods fail to accurately capture the economic value of physical assets in a competitive energy market (cf. extrinsic value)
- The real option valuation framework borrows the idea from classical financial option pricing theory and views a real asset or investment project as an option on the underlying cash flows.

Energy derivatives can be classified being “physical” of “financial” and regarding their complexity: more or less difficult task for pricing (assets market-dependent, “strategy”-dependent, etc.)

<table>
<thead>
<tr>
<th>PHYSICAL</th>
<th></th>
<th>FINANCIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>Exotic</td>
<td>Structured (Assets)</td>
</tr>
<tr>
<td>Future</td>
<td>Price-based</td>
<td>Volumetric</td>
</tr>
<tr>
<td>Forward</td>
<td>Asian option</td>
<td>Swing option</td>
</tr>
<tr>
<td>Swap</td>
<td>American option</td>
<td>Load serving contract</td>
</tr>
<tr>
<td>Spread</td>
<td>Swaption</td>
<td>Interruptible contract</td>
</tr>
<tr>
<td>European option</td>
<td>Tolling agreement</td>
<td>Storage facilities</td>
</tr>
</tbody>
</table>
Energy derivatives
Analytical or numerical methods for pricing derivatives?

- For pricing a derivative, one can have the choice between an analytical method or a numerical one.

- One has to make a trade-off for choosing an appropriate method for depending on the derivatives.

- Derivatives with CF depending only on market conditions: Forwards, Swaps, European options, Spread options, Load serving contracts, etc.
  - Prefer an analytical method
  - Absence de formule fermée

- Derivatives with CF depending on management strategies: Swing options, generating units, thermal assets, hydral assets, gas storages, etc.
  - Use a numerical method
  - Simplifications of the asset modelling
  - Optimisation needed

DATAS: Market conditions and Asset characteristics
MODEL: Model for market prices + Possible simplif. of the asset
Analytical method → Closed form solution (MtM, Delta, …)
Numerical method → Approximate solution (MtM, Delta, …)

Monte Carlo
EDP
Arbre
Quantif.
Monte Carlo
ENERGY DERIVATIVES

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Forwards in energy markets
Forwards can have a non standard delivery profile.

- Futures and forward contracts in energy markets
  - Firm agreement for purchase or sale of an underlying commodity over a certain future period in time, to a price fixed in advance, when the contract is made
  - No cash flow at the transaction date
  - Main characteristics are the delivery period and the power delivered (e.g. Base or Peak)
  - A forward has a given “profile”:

Buying an energy forward ⇔ Firm energy purchase contract over $[T_b, T_e]$
Selling an energy forward ⇔ Firm energy sale contract over $[T_b, T_e]$
Forwards and Swaps in energy markets

Swaps are a natural generalization of forward products

What are the cashflows generated by a forward contract with delivery on \([T_b, T_e]\) and profile \(Q\)? Deduce the fair fixed for this forward at time \(t\).

<table>
<thead>
<tr>
<th>Buying a forward contract (F(t, T_b, T_e))</th>
<th>Transaction date (t)</th>
<th>Delivery period ([T_b, T_e])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>For any (u \in [T_b, T_e]): (Q_u (S_u - F(t, T_b, T_e)))</td>
</tr>
<tr>
<td></td>
<td>(No cash)</td>
<td>(= ) Proportion of energy to the Spot ( - ) Forward price</td>
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\[
F(t, T_b, T_e) = \left( \sum_{u \in [T_b, T_e]} Q_u E_t[S_u] \right) / \left( \sum_{u \in [T_b, T_e]} Q_u \right)
\]

Swaps contracts are a natural generalization of forward products (forward ⇔ one-period swap), similar to swaps in financial markets

- Known as Contract for Differences (CfD) or Fixed for Floating forward contracts
- Swaps are flexible, OTC, easily customizable transactions
- Vanilla Swap (Contract for Differences) :
  Payment of a fixed predetermined price for a set of payments (financial) or deliveries (physical) at a sequence of dates in the future
- Various possibilities exist for the fixed and floating payments/deliveries...
Swaps in energy markets

Some practice...

In July 2013, a local gas distribution company wants to secure the base load supply of gas for the coming winter (October 2013 to March 2014) by a forward agreement.

The winter demand is 1 MWh/day. The forward prices observed on the market are the following:

<table>
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Swaps in €/MWh = \( \sum_{m} (Volume_{m} \times Price_{m}) / (\sum Volume_{m}) \)

Swap price = \( \frac{31 \times 26.5 + 30 \times 28 + \ldots + 31 \times 30.5}{31 + 30 + \ldots + 31} \)

Swap price = 29.9 €/MWh
Spreads in energy markets

Spreads are one of the most useful instrument in the “energy world”.

- Spread: price differential between two commodities viewed as 2 points of the energy price system.
- Spreads can be used to describe power plants, refineries, storage facilities, etc.

There are four classes of Spreads:
1. Intra-commodity spread
   - Ex. Spread on oils with different qualities
2. Inter-commodity spread: price differential between two different but related commodities
   - Between two operational inputs or two operational outputs or between inputs and outputs (processing spread)
   - Crack Spread: gasoline or heating oil (refined products) versus crude oil (input)
   - Spark Spread: electricity versus a primary fuel (gas, oil, fuel oil, uranium)
   - Dark Spread: electricity versus coal
   - Clean XY Spread: X versus Y compensated for the price of CO2
3. Time or calendar spread
   - Ex. Summer 2014 versus Winter 2014
4. Geographic spread: difference between prices of a same product in two different locations
**Spark and Dark spreads...**

**Practical application**

- We consider a gas-fired power plant with a 50% efficiency and emission costs of 0.20 ton CO\(_2\)/MWh and a coal-fired power plant with a 35% efficiency and emission costs of 0.30 ton CO\(_2\)/MWh.
- We assume following market conditions:

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a) Compute the Clean Spark and Clean Dark spreads. What do you deduce?

b) In these gas and coal market conditions, what is the threshold CO\(_2\) price conducing to switch from coal to gas power generation?
Spark and Dark spreads...

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a) Compute the Clean Spark and Clean Dark spreads. What do you deduce?

CSS = 50 – (30 + 0.20*6) / 0.5 = -12.4 €/MWh

CDS = 50 – (15 + 0.30*6) / 0.35 = 2 €/MWh

The gas-fired power plant is not profitable. We should better use coal-fired power plants.

b) In these gas and coal market conditions, what is the threshold CO$_2$ price conducing to switch from coal to gas power generation?
Spark and Dark spreads...

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**b)** In these gas and coal market conditions, what is the threshold CO\(_2\) price conducing to switch from coal to gas power generation?

CO\(_2\) price \(p^*\) such that: 50 – (30 + 0.20 \(p^*\)) / 0.5 = 50 – (15 + 0.30 \(p^*\)) / 0.35

\(\Rightarrow\) \(p^* = 37.5\) €/ton
Spark and Dark spreads...

Illustrate the theoretical benefits of using gas-fired or coal-fired plants for power generation

- Spark and Dark spreads strongly correlated \(\Rightarrow\) No-arbitrage condition between primary fuels
- Decoupling of these two Spreads \(\Rightarrow\) Loss of competitiveness of one of the two means of production

**Question**: What do you deduce from plots below?

**French Clean Dark and Clean Spark Spreads based on MAH forward prices (CRE, 2010)**

**German Clean Dark and Clean Spark Spreads based on YAH forward prices (CRE, 2011)**
Spark and Dark spreads...

Illustrate the theoretical benefits of using gas-fired or coal-fired plants for power generation

- Spark and Dark spreads strongly correlated ⇒ **No-arbitrage condition** between primary fuels
- Decoupling of these two Spreads ⇒ **Loss of competitiveness** of one of the two means of production

**Question**: What do you deduce from plots below?

- Beginning 2009 in France: coal-fired plants more rentable
- Germany from June 2011: anticipation of future use of coal-fired thermal power plant
- Negative Spark Spreads due to a higher level of gas prices

![French Clean Dark and Clean Spark Spreads based on MAH forward prices (CRE, 2010)](image1)

![German Clean Dark and Clean Spark Spreads based on YAH forward prices (CRE, 2011)](image2)
Transmission contracts
Valuation as commodity derivatives

- A transmission contract corresponds to the right to transfer a commodity from one point of the network to another point.

**Question**: How would you model such kind of contract?

![Diagram showing a transmission contract with points A and B, and the equation for commodity price: Commodity price at point A = $F^A$, Commodity price at point B = $F^B$, and the transport tariff $K$.]
Transmission contracts
Valuation as commodity derivatives

- A transmission contract corresponds to the right to transfer a commodity from one point of the network to another point.

- Question: How would you model such kind of contract?

This contract can be modelled as a geographic Spread option: option on the Spread A-B.

\[
MtM = \mathbb{E} \left[ \sum_{t \leq T} \left( F^A_t - F^B_t - K \right)^+ \right]
\]
ENERGY DERIVATIVES

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- Gas storage modelling
- Swing options pricing
- Load curve contract modelling
Power plants

Power generating units can be viewed as derivatives.

First approximation of a power plant market:

Pricing as a *Strip of calls on the spread* between the electricity price and its fuel price

\[
\text{MtM} = \mathbb{E} \left[ \sum_{t \leq T} \left( F_t^{\text{elec}} - h F_t^{\text{fuel}} \right)^+ \right]
\]

in which \([0, T]\) is the period of time considered and \(h\) is the *heat rate* of the power plant.

Depending on the fuel, this leads to so-called *spark spread* or *dark spread* options.

If log-normal dynamics are assumed for commodity prices, Margrabe closed formula (1978) for exchange options applies and provides value and Greeks.

Spread options can be used to model a large class of real derivatives:

- Other inter-commodity spread options: tolling agreements, refineries, etc.
- Calendar spread options: storage modelling, water reservoir
- Spread options on futures prices and spot prices are both important.
Power plants

More complex pricing methods are required to model more realistic power plants.

- However, this first approximation is not totally satisfactory... Why?

- Taking into account the CO₂ emission costs and additional operation costs:

Pricing as a *Strip of Clean fuel spreads*

\[
MtM = \mathbb{E} \left[ \sum_{t \leq T} \left( F_t^{\text{elec}} - h F_t^{\text{fuel}} - g F_t^{\text{CO₂}} - c \right)^+ \right]
\]

in which \( g \) is the proportional emission cost and \( c \) some operation costs (as start-up costs)

- Here, we still neglect *dynamic operational constraints* : ramp-up time (when they have just been started/turned off, power plants cannot be shut down/started up immediately), minimum power (arbitrage between stopping/restarting or running at Pmin), ...

- Additional constraints ➔ *Lower market value*

- Margrabe’s formula does not apply anymore...
  - Apart from previous simple case, *no closed-form solution* is known.
  - A first approximation consists in assuming the equivalent fuel cost log-normal...
  - Analytical methods rely on approximations, otherwise numerical approches (Monte-Carlo, ...) need to be use to solve such a *stochastic control problem.*
Spread options pricing

No closed formula exist for Spread options with non-zero strike.

- Spread option value decreases with substitution between fixed cost and initial input price.
- Pricing with different methods: approximations by Kirk, Eydeland and Carmona & Durrleman.
Spread options pricing
Margrable formula for a simple fuel spread options

- Consider the fuel spread option with maturity \( T \) priced at time \( t \):

\[
MtM = \mathbb{E}_t \left[ \left( F^e_T - hF^f_T \right)^+ \right]
\]

- Assume log-normal correlated dynamics for the electricity and fuel prices:

\[
\frac{dF^e_t}{F^e_t} = \sigma_e dW^e_t, \quad \frac{dF^f_t}{F^f_t} = \sigma_f dW^f_t \quad \text{with} \quad d\langle W^e, W^f \rangle_t = \rho dt
\]

- Margrabe provides the closed formula:

\[
MtM = F^e_t \mathcal{N}(d) - hF^f_t \mathcal{N}(d - \sqrt{V})
\]

\[
\begin{aligned}
N & \text{ is the c.d.f. of the Gaussian law and} \\
\begin{cases}
  d & = \frac{\ln(F^e_t/hF^f_t) + V/2}{\sqrt{V}} \\
  V & = (\sigma^2_e + \sigma^2_f - 2\rho\sigma_e\sigma_f)(T-t)
\end{cases}
\end{aligned}
\]

- Question: How does the option value vary when the electricity price increases? the fuel price increases?

- Question: What are the Deltas of this option?
Spread options pricing
Market value of a fuel spread option with Margrabe

Spread Option price (EUR)
Greeks of spread options
Sensitivities of fuel spread options to underlying factors

- **Deltas**: sensitivity to the different commodities
  - Long Delta: Sensitivity of the option to the electricity price
  - Short Delta: Sensitivity of the option to the fuel price

\[
\begin{align*}
\Delta_e &= \frac{\partial M_t M}{\partial F^e} = \mathcal{N}(d) \\
\Delta_f &= \frac{\partial M_t M}{\partial F^f} = -h \mathcal{N}(d - \sqrt{V})
\end{align*}
\]

Can be proven by using the key relation \( F^e_t f \mathcal{N}(d) = h F^f_t f \mathcal{N}(d - \sqrt{V}) \)

- **Vega**: sensitivity to the equivalent volatility elec-fuel: what is its sign?
Greeks of spread options

Sensitivities of fuel spread options to underlying factors

Deltas: sensitivity to the different commodities

- Long Delta: Sensitivity of the option to the electricity price
- Short Delta: Sensitivity of the option to the fuel price

\[
\begin{align*}
\Delta_e &= \frac{\partial M_{tM}}{\partial F_e} = N(d) \\
\Delta_f &= \frac{\partial M_{tM}}{\partial F_f} = -hN(d - \sqrt{V})
\end{align*}
\]

Can be proven by using the key relation

\[
F_t^e f_N(d) = h F_t^f f_N(d - \sqrt{V})
\]

Vega: sensitivity to the equivalent volatility elec-fuel: what is its sign?

\[
\nu = \left. \frac{\partial M_{tM}}{\partial \sigma_{eq}} \right| \geq 0
\]

Correlation Delta: sensitivity to the correlation between elec and fuel prices: what is its sign?
**Greeks of spread options**

**Sensitivities of fuel spread options to underlying factors**

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- **Vega**: sensitivity to the equivalent volatility elec-fuel: what is its sign?

\[ \nu = \frac{\partial \text{MtM}}{\partial \sigma_{eq}} \geq 0 \]

- **Correlation Delta**: sensitivity to the correlation between elec and fuel prices: what is its sign?

\[ \pi = \frac{\partial \text{MtM}}{\partial \rho} = -\nu \frac{\sigma_e \sigma_f}{\sigma_{eq}} \leq 0 \]
Greeks of spread options

Long Delta of a fuel spread option
Greeks of spread options
Sensitivity to the volatility of one of the two underlyings

- How does the fuel spread option value vary when the volatility of electricity prices increases?
- Due to (positive) correlation, it depends on the relative level of the electricity vol in comparison to the vol of fuel prices...

\[ \nu_e = \frac{\partial \text{MtM}}{\partial \sigma_e} = \nu \frac{\sigma_e - \rho \sigma_f}{\sigma_{eq}} \]

Spark spread option pricing: impact of Power prices volatility
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- **Gas storage modelling**
- Swing options pricing
- Load curve contract modelling
Storages

Storages are a major component of the gas chain

*Why using storage facilities?*

1. Physical and economic reasons (cost minimization)
   - Gas demand: variable (summer/winter) and inelastic
   - Inflexible gas supply: limited by the capacity of the pipeline system
   - Storages are closer to consumption areas
   - They ensure that gas will be easily accessible in response to higher demand
   - Interesting for non-producer countries with a high level of gas importation to reduce their dependance to producers

2. Regulation conditions
   - Gas supply companies have the obligation to own storage facilities to secure supply in periods of high demand

3. Financial reasons: arbitrage mechanism
   - Take benefit of the possible arbitrage between summer/winter or week-end/open days
   - Exploit market opportunities: inject gas in the storage while the gas is cheaper and withdraw it during periods with higher prices
   - Used for high consumption periods in gas but also for firing gas power plants (CCGT)
Gas storages

The storage has a strategic role for gas supply.

- Storage is used by supply companies to store an extra gas capacity (issued from production fields or importations).
- This buffer stock allows to hedge a part of the price risk (unexpected high demand).
- The storage has thus an effect on the summer-winter spread.

![Diagram showing the impact of gas storages on gas prices and supply over winter and summer.](#)
Gas storages

There are two main kinds of storage facilities (underground storages).

<table>
<thead>
<tr>
<th>Seasonal or base load storages</th>
<th>Fast-cycle or peak-load storages</th>
</tr>
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<tbody>
<tr>
<td><strong>Aquifers and depleted fields</strong></td>
<td><strong>Salt cavern: artificial cavity created by boring in salt rocks and dissolution</strong></td>
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**Diagram 1:**
- RESERVOIR SOUTERRAIN EN NAPPE AQUIFERE
- Injection
- Soutirage
- Gaz
- Eau
- Roche poreuse et perméable
- Couverture imperméable

**Diagram 2:**
- RESERVOIR SOUTERRAIN EN CAVITE SALINE
- Injection
- Soutirage
- Gaz
- Sel
- Argile et argile calcaire

- Large volume capacities but low injection/withdrawal rates (~10 months to totally fill and empty)
- Smaller reservoirs but high deliverability rates (around 2 weeks or less to fill and empty)
- Used to satisfy the **seasonal demand**: gas at lower price is stored in summer and delivered in winter
- Used for **arbitrage on short periods in time** (intraweek trading) and to respond to short-term demand
Gas storages

Optimal valuation of storages requires an optimization procedure.

- With the real option approach, the storage value is given by the maximal expected value that the storage scheduler can get by operating optimally the storage with respect to market conditions and stock constraints.
- An optimization is needed to determine the optimal gas volume to be injected/withdrawn, while respecting the constraints on the stock level.

Seasonal storage (180 days)  
Fast-cycle storage (20 days)
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- **Swing options pricing**
- Load curve contract modelling
Swing options in energy markets

Many physical or financial assets including an optionality can be considered as Swing options

**What is a Swing option?**

- Right to receive/purchase a given volume of energy
- Maximal number of ‘exercise’ rights \( n_{max} \) before the maturity
- The exchange price (strike) can be fixed, variable or random.
- Can be viewed as a *multiple-exercise American options*
- More complex Swing options include a variable volume: can be “swing up” or “swing down”
- In this case, global volume constraints impose a limited total energy amount

**Swing options** include a large class of structured assets in energy markets...

- Gas supply contracts including Take-or-Pay clauses
- Options to shut down services, demand-side mgmt contracts
- Storage facilities (but allow both injections and withdrawals)
- Water reservoirs, hydraulic assets (but include inflows)
Swing options with variable volume

Typical example of a gas Swing contract

- The buyer has the right to purchase each day a gas quantity in \([q_{\text{min}}, q_{\text{max}}]\)
- ... at a given strike price \(X\)
- ... but must purchase globally (over the year) a global volume in \([Q_{\text{min}}, Q_{\text{max}}]\)

- NB : The optimal strategy is non necessary bang-bang, meaning that the optimal consumption can belong to \([q_{\text{min}}, q_{\text{max}}]\)
Swing options pricing

The general pricing problem boils down to a stochastic control problem.

The market value of a variable volume Swing option is:

$$\text{MtM} = \sup_{(q_t)_{t \leq T}} \mathbb{E} \left[ \sum_{t \leq T} q_t (S_t - X_t) \right] q \text{ such that: } \begin{cases} q_t \in [q_{\text{min}}, q_{\text{max}}] \\ Q_T = \sum_{t \leq T} q_t \in [Q_{\text{min}}, Q_{\text{max}}] \end{cases}$$

Main difficulties:

- **Stochastic control problem** under constraints
- Numerical method needed (optimization required)
- Dimension of the problem: price(s) + volume
- Variable quantity $q$: bang-bang assumption $\Rightarrow$ multiple-exercise American option

Main idea in all numerical methods: use the **stochastic Dynamic Programming Principle (DDP)**

- Tree methods (binomial or trinomial) $\Rightarrow$ Leading to a forest of trees
- Monte Carlo techniques $\Rightarrow$ Longstaff-Schwartz (2001) for multiple-exercise American options
Swing options with variable volume

Typical example of a gas Swing contract: some practice...

Consider a gas Swing option over 180 days such that the minimal daily quantity is 10 MWh/day, the maximal daily quantity is 50 MWh/day and the global maximal quantity that can be purchased over the whole period is 6000 MWh.

After a standard normalization, this contract is usually separated into a firm Swap contract and a normalized Swing option.

a) What is the baseload volume provided by the Swap contract?
b) What is the remaining optional volume?
c) How many exercise rights are provided by the normalized Swing option?
d) Is the bang-bang assumption satisfied?

Swing with \( q \in [q_{\text{min}}, q_{\text{max}}] \) ⇔ Swap purchasing \( q_{\text{min}} \) every day + Purely Swing with \( q^* \in [0, q_{\text{max}} - q_{\text{min}}] \)
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\]

a) Baseload volume = 10 * 180 = \textbf{1800 MWh}
b) Remaining optional volume = 6000 – 1800 = \textbf{4200 MWh}
c) Number of exercise rights = 4200 / (50 – 10) = \textbf{105 times}
d) Yes, since there is an “integer” number of exercises in the Swing option
Swing options pricing
Reducing to multiple-exercise American options

- Swing contracts are decomposed in a firm Swap contract guaranteeing $q_{\text{min}}$ on each date + a purely Swing part normalized to purchases belonging to $[0, 1]$ with $n_{\text{max}}$ possible exercises.

- The discretization of admissible volumes is equivalent to the enumeration of possible exercises rights while respecting the constraints.
Swing options pricing

Some major properties of normalized Swing options market value

1. Case with only one exercise right: Swing option = American/Bermudan option

2. Upper bound (less constraint $\Rightarrow$ bigger value)
   \[
   \text{Swing option} \leq n_{\max} \times \text{Identical American options}
   \]

3. Lower bound (pre-determined choice of exercise dates $\Rightarrow$ lower value)
   \[
   \text{Swing option} \geq \sup_{N:n_{max} \text{ dates } \tau_k \leq T} \sum_{\tau_k \in N} \text{European options with maturity } \tau_k
   \]
Swing options pricing

Numerical method in practice

As multiple-exercise American options, Swing options can be valuate through:
- Backward induction in time
- An iteration on the number of exercise rights left

Number \( j \) of exercise rights left at time \( t_k \)?

Option value = 0

\[ j = 0 \]
\[ j > 0 \]

Current date?

Recursion in time

Option value = Payoff

\[ t_k = T \]
\[ t_k < T \]

Option exercise optimal?

Yes

No

Iteration on the number of exercise rights

Option value = \( j \) exercise rights left at time \( t_{k+1} \)

Option value = Payoff and \((j - 1)\) exercises left at time \( t_{k+1} \)
Swing options pricing

Dynamic Programming Principle

- Consider the problem of pricing a Swing option with \( j \) exercise rights on \( [t, T] \):

\[
\nu^{(j)}(t) = \sup_{t \leq \tau_1 < \ldots < \tau_j \leq T} \mathbb{E} \left[ \sum_{k=1}^{j} (S_{\tau_k} - \bar{X}_{\tau_k})^+ | \mathcal{F}_t \right]
\]

- Backward recursion on a discrete time grid
  Forward iteration on the number of exercise rights

- Mathematically, the Dynamic Programming Principle (DDP) for Swing options can be written:

\[
\begin{cases}
\nu^{(0)}(t_k) = 0, & \forall t_k \in \pi \\
\nu^{(j)}(T) = (S_T - \bar{X}_T)^+, & \forall j = 1, \ldots, n_{\text{max}} \\
\nu^{(j)}(t_k) = \max \{ \text{apply the } j^{\text{th}} \text{ exercise at } t_k; \text{ do not exercise at } t_k \} \\
= \max \{ (S_{t_k} - \bar{X}_{t_k})^+ + \mathbb{E} [\nu^{(j-1)}(t_{k+1}) | \mathcal{F}_{t_k}] ; \mathbb{E} [\nu^{(j)}(t_{k+1}) | \mathcal{F}_{t_k}] \} 
\end{cases}
\]

- As for American options, the main difficulty comes from the estimation of conditional expectations!
Swing options pricing

Using tree method...

- Idea: representing the price evolution on a tree

- Use of a binomial recombining tree
  - Possibility of upward and downward moves
  - At time $t_i$, the underlying price can take $(i + 1)$ values

\[ S_0 u^i, S_0 u^{i-1} d^1, \ldots, S_0 u^j d^{i-j}, \ldots, S_0 d^i \]

\[ S_{t_i}^{(i,j)} \quad q \quad \text{Up} \quad u S_{t_i}^{(i+1,j+1)} \]

\[ (1 - q) \quad \text{Down} \quad d S_{t_i}^{(i+1,j)} \]

\[ t_i \quad \Delta t \quad t_{i+1} \]
Example of trinominal recombining tree in a one factor model for gas prices:

Swing options pricing
Using tree method...
Swing options pricing

Using tree method...

- Assume the underlying price tree has been build.
- Backward recursion: the option price on node \((i, j)\) can be expressed as function of the option prices on nodes \((i+1, j+1)\) and \((i+1, j)\).

- Some recall for American options’ pricing...
  **DDP**: The option price at a node is the maximum between the exercise value (payoff at this node) and the continuation value (expected CF if the option is not exercised).

\[
v(i, j) = \max\{\mathbb{E}_{t_i} [v(t_{i+1})]; (S_{t_i} - \bar{X}_{t_i})^+\}
= \max\{q \cdot v(i + 1, j + 1) + (1 - q) \cdot v(i + 1, j); (S_{t_i} - \bar{X}_{t_i})^+\}
\]

- Swing options’ case: it leads to a forest of trees (number of trees = number of exercise rights)!

\[
v^{(0)}(i, j) = 0
v^{(k)}(i, j) = \max\{\mathbb{E}_{t_i} [v^{(k)}(t_{i+1})]; (S_{t_i} - \bar{X}_{t_i})^+ + \mathbb{E}_{t_i} [v^{(k-1)}(t_{i+1})]\}
= \max\{q \cdot v^{(k)}(i + 1, j + 1) + (1 - q) \cdot v^{(k)}(i + 1, j);
(S_{t_i} - \bar{X}_{t_i})^+ + q \cdot v^{(k-1)}(i + 1, j + 1) + (1 - q) \cdot v^{(k-1)}(i + 1, j)\}
\]
ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- Gas storage modelling
- Swing options pricing
- Load curve contract modelling
Load serving contract
A demand-side typical contract in energy portfolios

- The supplier is engaged to deliver some energy whatever the consumption.
- Concerns only customers, whose consumption is precisely known (load curve).
- Different prices depending on the real consumption being inside, below or above some tunnel.