PhD Project- Traffic state Reconstruction and Traffic flow Management

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Working Context. The PhD will be officially co-supervised by Thibault Liard (Maître de Conférences, XLIM, Limoges) and Pierre Lissy (Junior Professor, HDR, CERMICS, École des Ponts ParisTech), and also co-advised by Amaury Hayat (Researcher and Professor, HDR, CERMICS, École des Ponts ParisTech.

Scientific context and positioning. The advent of vehicle automation has the potential to substantially transform the management of transportation systems. As disruptive technologies such as *autonomous vehicles* (AVs) come closer to reality, the potential for improved traffic control using AVs as controllers on the traffic flow has become a significant focus both theoretically [11] and experimentally [12].

Such control strategies often rely on traffic models to capture the dynamics of the real traffic flow. The traffic flow at the macroscopic scale can be modeled by some particular partial differential equations (PDEs) under the form of conservation laws:

$$\partial_t \rho(t,x) + \partial_x f(\rho(t,x)) = 0,\ (t,x) \in \mathbb{R}_+^* \times [0,L] \tag{1}$$

The function $(t,x) \mapsto \rho(t,x)$ stands for the density of cars at time t and at the position x. Both theoretically and numerically, this family of equations is rather peculiar among all PDEs since one can show that for many smooth initial data ρ_0, the solution develops discontinuities in finite time and the global (in time) existence of classical solutions fails. Considering weak solutions is quite natural, but uniqueness is lost and one finds most of those weak solutions to be wildly physically irrelevant to underlying physical situations. To model the impact of AVs on the traffic flow, one needs to select a weak solution using a flow condition at the location of the AV [8, 7]. This weak solution of (1) is "non-classical", and in particular different from the weak solution inspired by physics often bear the name of entropy solutions [4], but more relevant in the context of traffic flow.

There exists a large variety of controllers dealing with stabilization of hyperbolic system as Proportional controller, Proportional-Integral controller [5] or Sliding Mode Controller [6]. These laws require a good and accurate knowledge of road density ρ in (1). In [10, 13], the mass of the weak-entropy solution of (1) over the segment $[0,L]$ is needed to stabilize (1) around a stationary point. There comes the need for an efficient state reconstruction algorithm using measurements from fixed sensors but also capable of dealing with GPS measurements. In [1], the authors consider the problem of traffic density reconstruction using measurements from a small number of probe vehicles. The model used assumes noisy measurements and a partially unknown first-order model. All these considerations make the use of machine learning to reconstruct the state the only applicable solution. However, these reconstruction techniques suffer from a lack of theoretical guarantee and need a large amount of measurements.

Now, to implement control strategies for conservation laws, there is a huge literature dealing with asymptotic stabilization of hyperbolic systems for regular solutions (see the book [2]). However, to our knowledge, there are no theoretical results for the stabilization of conservation laws for the "non-classical" weak solutions described above. In the context of traffic flow, the recent articles [11, 3, 9] have numerically attempted to control the trajectory of AVs to have a positive impact on the traffic flow. However, these controllers suffer from a lack of theoretical guarantee, automation and robustness.

Overview of the project. In this PhD thesis, we will design systematic and robust controllers that stabilize system of conservation laws (1) using traffic state reconstruction and machine learning algorithms. This will be done by addressing the following two problems:

1. Traffic state reconstruction using the residual neural network (ResNets) and noisy measurements of the initial and final positions of few probes. Moreover, theoretical guarantees will be obtained using statistical analysis, neural ODE and derivation of macroscopic models from microscopic models.

2. Stabilization of conservation laws (1) using feedback controllers from the automatic community depending on the reconstructed traffic state using the residual neural network (ResNets). Those stabilization results will be validated theoretically by extending the notion of generalized characteristics.
3. Stabilization of conservation laws (1) using feedback controllers obtained from machine learning techniques such as Reinforcement Learning (RL).

This project will be carried out by a synergy of combined expertise in **machine learning**, **optimization**, **control theory** and **dynamic models**.

References

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