# Calibration of the Libor Market Model with Jacobi stochastic volatility factor

## Insurance practices

Sophian MEHALLA[1,2]
Joint work with P.-E. Arrouy[2], A. Boumezoued[2] & B. Lapeyre[1]

[1]École Nationale des Ponts et Chaussées (CERMICS)
& [2]Milliman Paris

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# Regulatory framework (1/2)

- European regulatory framework: Solvency II (2016)

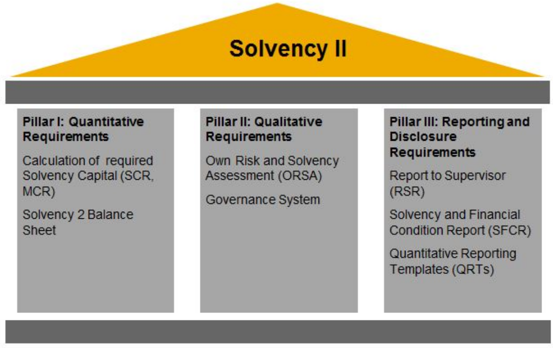


Figure: Solvency II regulatory framework

# Regulatory framework (1/2)

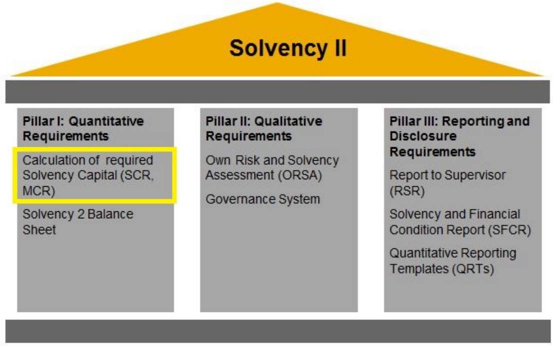


Figure: Solvency II regulatory framework

→ Focus on the computation of the SCR.

# Regulatory framework (2/2)

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- Insurers are exposed to a lot of risks: behaviour of policyholders, mortality rates, (natural) disasters, operational risks, financial risk, . . .
- Solvency Capital Requirement (SCR) is a "value" of all these risks.

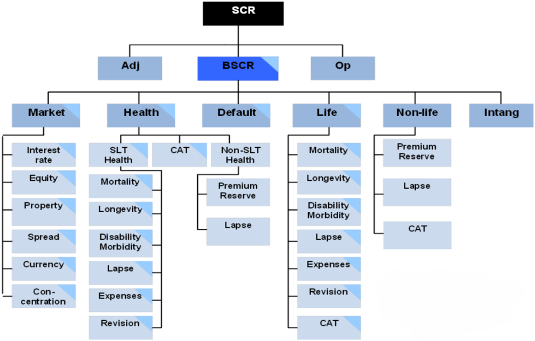


Figure: Structure of the standard formula

# Regulatory framework (2/2)

- Among all these risks: the **financial risk** (rise/fall of interest rates, fall of some stocks, etc.).

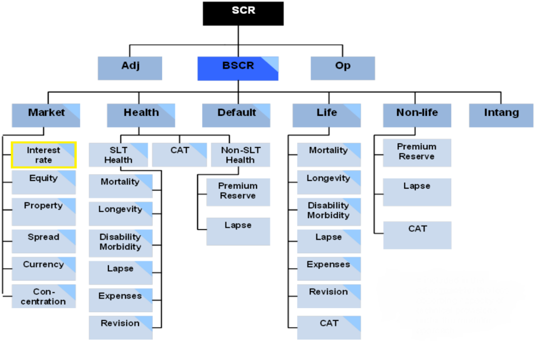


Figure: Structure of the standard formula

→ Major part of insurers' portfolio are composed of *bonds* (≈ 80%) and other derivatives on *interest-rates*.

# What is required?

- **Models dedicated to interest rates are decisive**, some may be complex to handle. They are asked..
  - ..to be *Risk-Neutral*;
  - ..to be consistent with market data (*market-consistency*): models have to replicate market prices.
- Models from the bank industry have been chosen.

# Why stochastic volatility models?

Goal: price swaptions (call option on swap rate $S_T^{m,n}$)

$$\text{Price}(\sigma^{\text{implied}}; K, T) = \text{Discount} \times \mathbb{E}^S\left[\max(S_T^{m,n} - K, 0)\right]$$

$\rightsquigarrow$ Choice: model the financial driver swap rate using an Ito diffusion. The use of stochastic volatility type models is especially adapted.

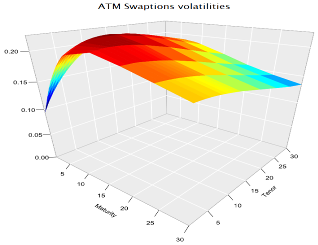


Figure: Market data to be replicated

Standard uses: $\approx$ 10 parameters to fit around 300 (swaption) prices.

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# Zero-Coupon bond

The LIBOR Market Model focuses on the modelling of observable quantities (following the work of (BGM97) and (Jam97); see (BM07) for an overview of interet-rates modelling). Let $T > 0$ be a finite time horizon, and let us assume:

- the market information is generated by a $N$-dimensional Brownian motion $(W_t)_{t \leqslant T}$;
- there exists a Risk-Neutral probability $\mathbb{P}^*$ (equivalent to the historical one) under which discounted bond prices are martingales.

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$$\rightsquigarrow \text{Under } \mathbb{P}^*, \ \frac{dP(t,T)}{P(t,T)} = r_t dt + \sigma(t,T) \cdot dW_t^*$$

with

- $(r_t)_{t \leqslant T}$ is the risk-free rate;
- $(\sigma(t,T))_{t \leqslant T}$ is the volatility structure (adapted process);
- the correlation between Zero-Coupon bonds $P(t,T)$ and its volatility structure $\sigma(t,T)$ can be identified.

# Forward rates

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- Consider a tenor structure $T_0 < T_1 < \cdots < T_K \leqslant T$. For $k \in [\![0, K-1]\!]$, the forward rate prevailing over the period $[T_k, T_{k+1}]$ is defined by:

$$F_k(t) := \frac{1}{\Delta_k}\left(\frac{P(t,T_k)}{P(t,T_{k+1})} - 1\right),\ t \leqslant T_k,\ \Delta_k := T_{k+1} - T_k.$$

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- Under $\mathbb{P}^*$, the dynamic of these is (Ito):

$$dF_k(t) = F_k(t)\gamma_k(t) \cdot \left(dW_t^* - \sigma(t, T_{k+1})dt\right)$$

where $\gamma_k(t) := \frac{1+\Delta_k F_k(t)}{\Delta_k F_k(t)}\left(\sigma(t,T_k) - \sigma(t,T_{k+1})\right)$.

# Shifted forward rates

- "Late" market conditions have been such that these rates could be negative: hence the introduction of a *shift* coefficient $\delta \geqslant 0$. Our new modelling framework (see (JR03)) focuses on the *shifted forward rates*

$$F_k(t) + \delta,\ t \leqslant T_k$$

and is such that

$$\mathbb{P}^* \left( \forall t \leqslant T_k : F_k(t) \geqslant -\delta \right) = 1.$$

- Under $\mathbb{P}^*$, the dynamic of shifted rates is *assumed* to be:

$$dF_k(t) = (F_k(t) + \delta)\gamma_k(t) \cdot \left( dW_t^* - \sigma(t, T_{k+1})dt \right).$$

# Swap rate

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$$S_t^{m,n} := \frac{P(t,T_m) - P(t,T_n)}{B^S(t)}, \ t \leqslant T_m,$$

with $B^S(t) := \sum_{j=m}^{n-1} \Delta_j P(t, T_{j+1})$.

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$$\left(S_t^{m,n} + \delta\right)_{t \leqslant T_m}.$$

- It can be shown that the shifted swap rate expresses as a deterministic function of the shifted forward rates involved during the time interval $[T_m, T_n]$.
- Under $\mathbb{P}^S$, the probability measure associated to the *numéraire* $B^S(t)$, the dynamic of the shifted swap rate is (Ito):

$$d(S_t^{m,n} + \delta) = \sum_{j=m}^{n-1} \frac{\partial (S_t^{m,n} + \delta)}{\partial (F_j(t) + \delta)} \left(F_j(t) + \delta\right) \gamma_j(t) \cdot dW_t^S$$

  where the quantities $\partial (S_t^{m,n} + \delta) / \partial (F_j(t) + \delta)$ can be analytically computed.

# The volatility component

- One of the most popular choice is

$$\sigma(t,T) = v(t,T) \times \sqrt{V_t}$$

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with

- $(t,T) \longmapsto v(t,T)$ a deterministic function (bounded and piecewise continuous);
- $(V_t)_{t \leqslant T}$ a Cox-Ingersoll-Ross process. Its dynamic is usually specified under the Risk-Neutral measure and the dynamic under $\mathbb{P}^S$ is deduced thanks to Girsanov's theorem:

$$dV_t = \kappa(\theta - \xi(t)V_t)dt + \epsilon\sqrt{V_t}dZ_t^S$$

which Feller condition $2\kappa\theta \geqslant \epsilon^2$ ensures to have $\mathbb{P}^*\left(\forall t \leqslant T : V_t > 0\right) = 1$ (as long as $V_0 > 0$);

  - $t \longmapsto \xi(t)$ is a function appearing through the change of measures: it depends on the forward rates $(F_j(t))_{t \leqslant T_j, j \in \{0,\cdots,n-1\}}$.

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# Calibration of the model

- The model is, under $\mathbb{P}^S$:

$$\begin{cases} d(S_t^{m,n} + \delta) = \sqrt{V_t} \sum_{j=m}^{n-1} \frac{\partial (S_t^{m,n} + \delta)}{\partial (F_j(t) + \delta)} (F_j(t) + \delta) \eta_j(t) \cdot dW_t^S \\ dV_t = \kappa(\theta - \xi(t) V_t) dt + \epsilon \sqrt{V_t} dZ_t^S \\ (S_0^{m,n} + \delta, V_0) \in \mathbb{R} \times \mathbb{R}_+^* \end{cases}$$

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# Calibration of the model

- The model is, under $\mathbb{P}^S$:

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- As it stands, the model is too complex to be calibrated: Monte-Carlo simulations are out of the operational scope.
- Based on the assumption of low variability of some ratios, these are *freezed* to their initial value.

# Calibration of the model: *freezing* technique

- Two ways of approximating the dynamic:
  - the "log-normal" version (Heston type model)

$$\begin{cases} d(S_t^{m,n}+\delta) = \sqrt{V_t}(S_t^{m,n}+\delta)\sum_{j=m}^{n-1}\frac{\partial(S_0^{m,n}+\delta)}{\partial(F_j(0)+\delta)}\frac{F_j(0)+\delta}{S_0^{m,n}+\delta}\eta_j(t)\cdot dW_t^S \\ dV_t = \kappa(\theta-\xi^0(t)V_t)dt+\epsilon\sqrt{V_t}dZ_t^S \\ (S_0^{m,n}+\delta, V_0)\in\mathbb{R}_+\times\mathbb{R}_+^* \end{cases}$$

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  with $t\rightarrow\xi^0(t)$ a deterministic (bounded and piecewise continuous) function.
- The second model is more suited under low interest-rates regime: we will focus on this version of the model in the following.

# Calibration using Gram-Charlier expansion

- To calibrate (1), we chose to perform a Gram-Charlier expansion, following the work of (ABBD17), on the unkown density, $f$, of $S^{m,n}_{T_m}$: the model is calibrated thanks to approximating prices.

$$\text{Price} = \int_{\mathbb{R}} (s-K)_+ f(s)ds \approx \int_{\mathbb{R}} (s-K)_+ f^{(N)}(s)ds =: \text{Price}(N)$$

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- Is this approximation accurate?

# Theory of Gram-Charlier expansions

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- If $f/g \in L^2(g)$,

$$\frac{f^{(N)}}{g} \xrightarrow[N\to\infty]{L^2(g)} \frac{f}{g},$$

with the approximating densities $f^{(N)}(x) := g(x) \times \sum_{i=0}^{N} c_i H_i(x)$.

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  with the approximating densities $f^{(N)}(x) := g(x) \times \sum_{i=0}^{N} c_i H_i(x)$.
- The coefficients $c_i$ are linear combinations of the moments of $f$.
- $f/g \in L^2(g)$ means $\int_{\mathbb{R}} f(x)^2 e^{\frac{x^2}{2}} dx < \infty$

# Gram-Charlier and stochastic volatility models (1/2)

$$X := \sqrt{V} \times G$$

with $G \sim N(0,\sigma^2)$ and $V \sim \chi^2(d)$, $G$ and $V$ being independent.

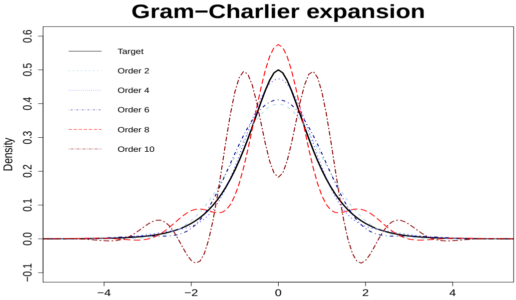


Figure: Gram-Charlier expansion of the density of $X$ up to order 10 - $\sigma^2 = 0.25$ & $d = 4$

Unbounded volatility processes: Gram-Charlier expansion unlikely to converge

# Gram-Charlier and stochastic volatility models (2/2)

$$X^{(M)} := \sqrt{\min(V, M)} \times G$$

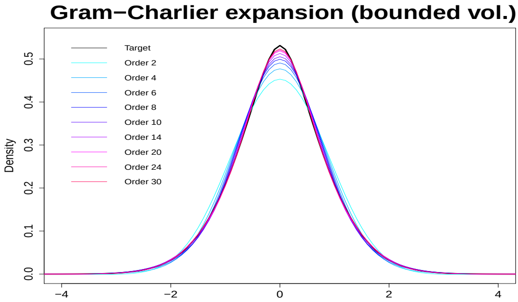


Figure: Gram-Charlier expansion of the density of $X^{(M)}$ up to order 30 - $M = 4$

Requirement: $\sigma^2 M < 2$

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# Proposed dynamic (1/2)

- Modelling assumption: the tails of (marginal) distribution of the volatility process in (1) are thin.
- We bound the volatility process to perform Gram-Charlier expansion, while preserving (we hope!) a good approximation of the distribution.

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- Modelling assumption: the tails of (marginal) distribution of the volatility process in (1) are thin.
- We bound the volatility process to perform Gram-Charlier expansion, while preserving (we hope!) a good approximation of the distribution.
- Let us define a bounding function

$$Q(v) = \frac{(v - v_{\min})(v_{\max} - v)}{(\sqrt{v_{\max}} - \sqrt{v_{\min}})^2}$$

is such that $Q(v) \in [v_{\min}, v_{\max}]$ for all $v \in [v_{\min}, v_{\max}]$. Note that $Q(v) \to v$ as $(v_{\min}, v_{\max}) \to (0, \infty)$.

# Proposed dynamic (2/2)

- Based on the work (AFP18), we introduce the Jacobi dynamic for the stochastic volatility component:

$$\begin{cases} d(S_t^{m,n}+\delta) = \rho(t)\sqrt{Q(V_t)}\|\lambda(t)\| \times dZ_t^S + \sqrt{V_t - \rho(t)^2 Q(V_t)}\lambda(t)\cdot dZ_t^{S,\perp} \\ dV_t = \kappa(\theta - \xi^0(t)V_t)dt + \epsilon\sqrt{Q(V_t)}dZ_t^S \\ (S_0^{m,n}+\delta, V_0) \in \mathbb{R}\times ]v_{\min}, v_{\max}[ \end{cases} \quad (2)$$

with

- $\lambda(t) := \sum_{j=m}^{n-1} \frac{\partial(S^{m,n}(0)+\delta)}{\partial(F_j(0)+\delta)}(F_j(0)+\delta)\eta_j(t)$;
- $\rho(t)$ represents the correlation structure between the swap rate and its stochastic volatility.

# Some properties

- Roughly
$$d(S_t^{m,n}+\delta)\approx\sqrt{V_t}\lambda(t)\cdot dW_t^S$$
- While (1) is an affine model, (2) is a polynomial model which allow to compute marginal moments of $S^{m,n}$ (see (CKRT12) or (FL16) for more details on the polynomial processes).

# Some properties

- Roughly
$$d(S_t^{m,n}+\delta)\approx\sqrt{V_t}\lambda(t)\cdot dW_t^S$$
- While (1) is an affine model, (2) is a polynomial model which allow to compute marginal moments of $S^{m,n}$ (see (CKRT12) or (FL16) for more details on the polynomial processes).
- If Feller's condition $\frac{\epsilon^2(v_{max}-v_{min})}{(\sqrt{v_{max}}-\sqrt{v_{min}})^2}\leqslant 2\kappa\min(v_{max}-\theta,\theta-v_{min})$ holds,
$$\mathbb{P}^*\left(\forall t: V_t\in]v_{\text{min}},v_{\text{max}}[\right)=1.$$
- Then, Gram-Charlier expansion can be performed on the unknown density of $S_T^{m,n}$, as long as:
$$v_{max}\times T\times\max_{t\leqslant T}\|\lambda(t)\|^2<2$$

# Convergence towards the reference dynamic (1/2)

- Weak convergence of solution to (2) towards solution of (1) as $(v_{\min}, v_{\max}) \to (0, \infty)$ is shown in (AFP18).

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- We have more:

**Theorem**

*Fix $v_{\min} = 0$. There exists finite constants $C, K$ such that*

$$\sup_{0 \leqslant t \leqslant T} \mathbb{E}^* \left[ |V_t^{Jacobi} - V_t| \right] \leqslant C / \log(v_{max}),$$

*and*

$$\mathbb{E}^* \left[ \sup_{0 \leqslant t \leqslant T} |V_t^{Jacobi} - V_t| \right] \leqslant K / \sqrt{\log(v_{max})}.$$

# Convergence towards the reference dynamic (2/2)

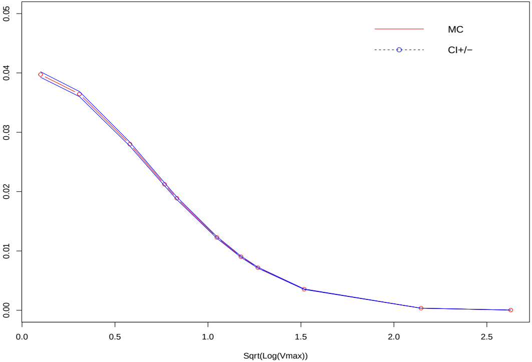


Figure: $\mathbb{E}\left[\sup_{0\leqslant t\leqslant 5}|V_t^{\text{Jacobi}} - V_t|\right]$ obtained by Monte-Carlo simulations

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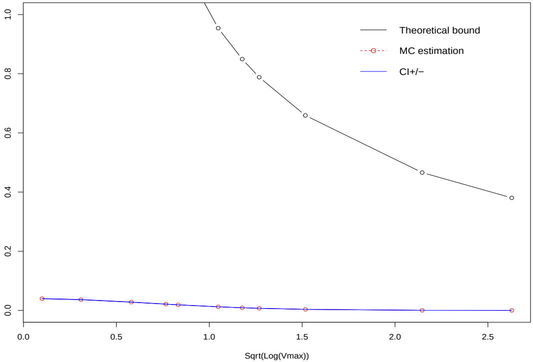


Figure: $\mathbb{E}\left[\sup_{0 \leqslant t \leqslant 5} |V_t^{\text{Jacobi}} - V_t|\right]$ and theoretical bound

# Conclusion & Perspectives

- General theoretical framework allowing to perform a convergent Gram-Charlier expansion..

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- Impact on the SCR?

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# End of presentation

Thank you!

(sophian.mehalla@enpc.fr)