Optimization of Energy Production and Transport

A stochastic decomposition approach

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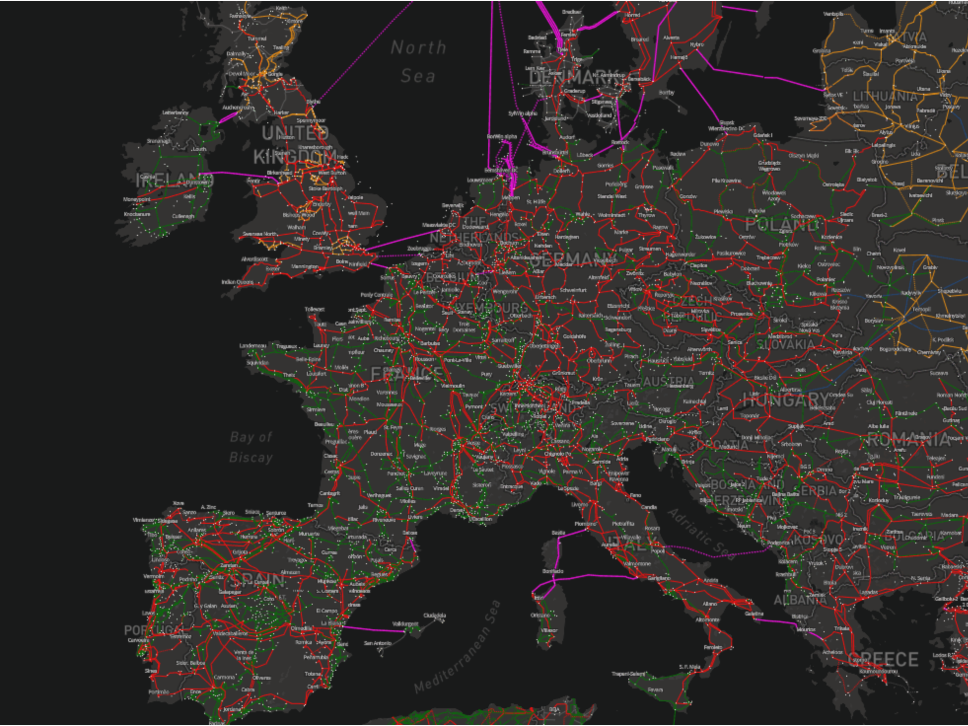
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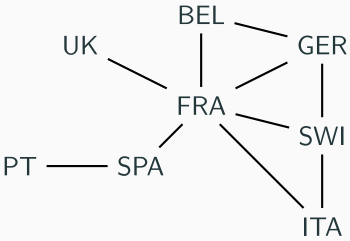
Motivation

An energy production and transport optimization problem on a grid modeling energy exchange across European countries.[1]



- Stochastic dynamical problem.
- Discrete time formulation (weekly or monthly time steps).
- Large-scale problem (8 countries).

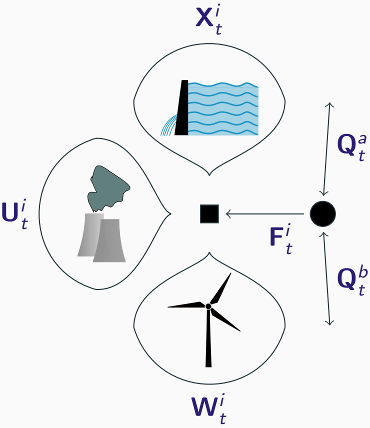
[1]But the framework remains valid for smaller energy management problems.

Lecture outline

Modelling

Production at each node of the grid

At each node i of the grid, we formulate a production problem on a discrete time horizon $[\![0, T]\!]$, involving the following variables at each time t:



- $\mathbf{X}_t^i$: state variable (dam volume)
- $\mathbf{U}_t^i$: control variable (energy production)
- $\mathbf{F}_t^i$: grid flow (import/export from the grid)
- $\mathbf{W}_t^i$: noise (consumption, renewable)

Writing the problem for each node

For each node $i \in [\![1, N]\!]$:

- The dynamic $x_{t+1}^i = f_t^i(x_t^i, u_t^i, w_t^i)$ writes

$$x_{t+1}^i = x_t^i + \underbrace{a_t^i}_{\text{inflow}} - \underbrace{p_t^i}_{\text{turbinate}} - \underbrace{s_t^i}_{\text{spillage}} \; .$$

- The load balance (supply $=$ demand) gives

$$\underbrace{p_t^i}_{\text{turbinate}} + \underbrace{g_t^i}_{\text{thermal}} + \underbrace{r_t^i}_{\text{recourse}} + \underbrace{f_t^i}_{\text{grid flow}} = \underbrace{d_t^i}_{\text{demand}} \; .$$

Thus, we explicit w_t^i and u_t^i:

$$w_t^i = (a_t^i, d_t^i) \; , \;\; u_t^i = (p_t^i, s_t^i, g_t^i, r_t^i) \; .$$

We pay to use the thermal power plant and we penalize the recourse:

$$L_t^i(x_t^i, u_t^i, f_t^i, w_t^i) = \underbrace{\alpha_t^i (g_t^i)^2 + \beta_t^i g_t^i}_{\text{quadratic cost}} + \underbrace{\kappa_t^i r_t^i}_{\text{recourse penalty}} \; .$$

A stochastic optimization problem decoupled in space

At each node i of the grid, we have to solve a stochastic optimal control subproblem depending on the grid flow process $\mathbf{F}^i$:[2]

$$J^i_{\mathfrak{P}}[\mathbf{F}^i] = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}^i_{t+1}) + K^i(\mathbf{X}^i_T) \Big) ,$$

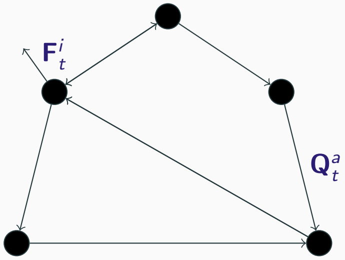
$$\begin{aligned} \text{s.t.} \quad & \mathbf{X}^i_{t+1} = f^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}^i_{t+1}) , \\ & \mathbf{X}^i_t \in \mathcal{X}^{i,\mathrm{ad}}_t , \quad \mathbf{U}^i_t \in \mathcal{U}^{i,\mathrm{ad}}_t , \\ & \mathbf{U}^i_t \preceq \mathcal{F}_t , \end{aligned}$$

The last equation is the measurability constraint, where $\mathcal{F}_t$ is the σ-field generated by the noises $\{\mathbf{W}^i_\tau\}_{\tau=1\dots t}$ up to time t.

[2] The notation $J^i_{\mathfrak{P}}[\cdot]$ means that the argument of $J^i_{\mathfrak{P}}$ is a *random variable*.

Modeling exchanges between countries

The grid is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$. At each time $t \in [\![0, T-1]\!]$ we have:



- a flow $\mathbf{Q}_t^a$ through each arc a, inducing a cost $c_t^a(\mathbf{Q}_t^a)$, modeling the exchange between two countries
- a grid flow $\mathbf{F}_t^i$ at each node i, resulting from the balance equation

$$\mathbf{F}_t^i = \sum_{a \in input(i)} \mathbf{Q}_t^a - \sum_{b \in output(i)} \mathbf{Q}_t^b$$

A transport cost decoupled in time

At each time step $t \in [\![0, T-1]\!]$, we define the transport cost as the sum of the cost of the flows $\mathbf{Q}_t^a$ through the arcs a of the grid:

$$J_{\mathfrak{T},t}[\mathbf{Q}_t] = \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a)\Big) ,$$

where the c_t^a's are easy to compute functions (say quadratic).

Kirchhoff's law

The balance equation stating the conservation between $\mathbf{Q}_t$ and $\mathbf{F}_t$ rewrites in the following matrix form:

$$A\mathbf{Q}_t + \mathbf{F}_t = 0 ,$$

where A is the node-arc incidence matrix of the grid.

The overall production transport problem

The *production cost* $J_{\mathfrak{P}}$ aggregates the costs at all nodes i:

$$J_{\mathfrak{P}}[\mathbf{F}] = \sum_{i \in \mathcal{N}} J^i_{\mathfrak{P}}[\mathbf{F}^i] \ ,$$

and the *transport cost* $J_{\mathfrak{T}}$ aggregates the costs at all time t:

$$J_{\mathfrak{T}}[\mathbf{Q}] = \sum_{t=0}^{T-1} J_{\mathfrak{T},t}[\mathbf{Q}_t] \ .$$

The compact production-transport problem formulation writes:

$$\min_{\mathbf{Q},\mathbf{F}} \quad J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \qquad \text{(P)}$$

$$\text{s.t.} \ \ A\mathbf{Q} + \mathbf{F} = 0 \ .$$

Resolution methods

Where are we heading to?

The problem P has:

- N nodes (with $N = 8$);
- T time steps (with $T = 12$ or $T = 52$);
- N independent random variables per time step t: $\mathbf{W}_t^1, \cdots, \mathbf{W}_t^N$.

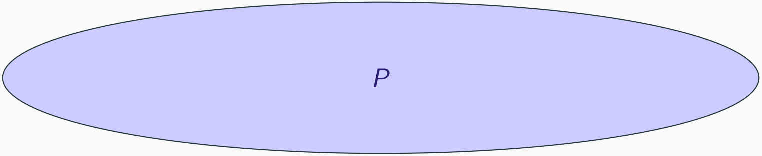
We aim to solve the problem numerically. We suppose that for all t, $\mathbf{W}_t^i$ is a discrete random variable, with support size $\mathfrak{n}_{bin}$. Thus, the random variable

$$\mathbf{W}_t = (\mathbf{W}_t^1, \cdots, \mathbf{W}_t^N) ,$$

has a support size $\mathfrak{n}_{bin}^N$ (because of the independence).

First idea: solving the whole problem inplace!

Write the problem and solve it!



But ...

- N nodes and T time steps.
- Non-anticipativity constraint: we ought to formulate the problem on a tree (Stochastic Programming approach)

$$\text{number of nodes} = (\mathfrak{n}_{bin}^{N})^{T} = \mathfrak{n}_{bin}^{NT} ,$$

giving a complexity in $\mathcal{O}(\mathfrak{n}_{bin}^{NT})$.

The problem is not tractable ...

Second idea: decomposition with Dynamic Programming

We assumed that the noise $\mathbf{W}_0, \cdots, \mathbf{W}_T$ were independent.
We decompose the problem time step by time step $\rightarrow$ T subproblems

P :

The complexity reduces to $\mathcal{O}(T\mathfrak{n}_{bin}^N)$. We use Dynamic Programming to compute the value functions $V_1, \cdots, V_T$.

But ...

- N nodes: curse of dimensionality
- Still a support size $\mathfrak{n}_{bin}^N$ for $\mathbf{W}_t$

We use Stochastic Dual Dynamic Programming to solve the problem with $N = 8$ dimensions.

A brief recall on Stochastic Dynamic Programming

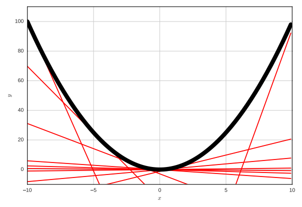
Dynamic Programming

We compute value functions with the backward equation:

$$V_T(x) = K(x)$$

$$V_t(x_t) = \min_{u_t} \mathbb{E}\Big[\underbrace{L_t(x_t, u_t, \mathbf{W}_{t+1})}_{\text{current cost}} + \underbrace{V_{t+1}\Big(f(x_t, u_t, \mathbf{W}_{t+1})\Big)}_{\text{future costs}}\Big]$$

Stochastic Dual Dynamic Programming



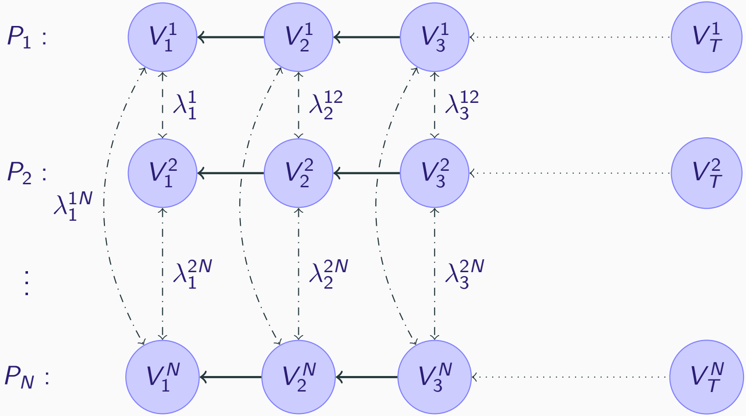
- Convex value functions V_t are approximated as a supremum of a finite set of affine functions
- Affine functions (=cuts) are computed during forward/backward passes, till convergence

$$\widetilde{V}_t(x) = \max_{1 \leq k \leq K} \{\lambda_t^k x + \beta_t^k\} \leq V_t(x)$$

- SDDP makes an extensive use of LP/QP solver

Third idea: spatial decomposition

We decompose the problem time by time and node by node to obtain $N \times T$ decomposed subproblems:



The complexity reduces to $\mathcal{O}(NT\mathfrak{n}_{bin})$! But ...

How to compute the different λ?

Introducing decomposition methods

The decomposition/coordination methods we want to deal with are iterative algorithms involving the following ingredients.

- Decompose the global problem in several subproblems of smaller size by dualizing the constraint $A\mathbf{Q} + \mathbf{F} = 0$,
- Coordinate at each iteration the subproblems using the price λ.

$$A\mathbf{Q} + \underbrace{\mathbf{F}}_{\textit{allocation}} = 0 \quad \rightsquigarrow \underbrace{\boldsymbol{\lambda}}_{\textit{price}}$$

- Solve the subproblems using Dynamic Programming, taking into account the price transmitted by the coordination.

Approximating the subproblems

In both cases, the subproblems encompass a new "noise", that is, the price multiplier $\boldsymbol{\lambda}_t^{(k)}$, which may be correlated in time.
The white noise assumption fails.

Dynamic Programming cannot be used for solving the subproblems.

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In order to overcome this difficulty, we use a trick that involves approximating the new noise $\boldsymbol{\lambda}_t^k$ by its conditional expectation w.r.t. a chosen random variable $\mathbf{Y}_t$.

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In order to overcome this difficulty, we use a trick that involves approximating the new noise $\boldsymbol{\lambda}_t^k$ by its conditional expectation w.r.t. a chosen random variable $\mathbf{Y}_t$.

Assume that the process $\mathbf{Y}$ has a given dynamics:

$$\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1}) \ .$$

If noises $\mathbf{W}_t$'s are time independent, then $(\mathbf{X}_t^i, \mathbf{Y}_t)$ is a valid state for the i-th subproblem and Dynamic Programming applies.

Price decomposition

The production and transport optimization problem writes

$$\min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \qquad \text{s.t.} \quad A\mathbf{Q} + \mathbf{F} = 0 \,. \qquad (\mathcal{P})$$

The decomposition scheme consists in dualizing the constraint, and then in approximating the multiplier $\boldsymbol{\lambda}$ by its conditional expectation w.r.t. $\mathbf{Y}$. This trick leads to the following problem

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}(\boldsymbol{\lambda} \mid \mathbf{Y}) \,, A\mathbf{Q} + \mathbf{F} \right\rangle \,.$$

It is not difficult to prove that this dual problem is associated to the following relaxed primal problem:

$$\min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \qquad \text{s.t.} \quad \mathbb{E}\big(A\mathbf{Q} + \mathbf{F} \;\big|\; \mathbf{Y}\big) = 0 \,,$$

and hence provides a lower bound of $(\mathcal{P})$.

A dual gradient-like algorithm

Applying the Uzawa algorithm to the dual problem

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{Q},\mathbf{F}} \; J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}(\boldsymbol{\lambda} \mid \mathbf{Y}), A\mathbf{Q} + \mathbf{F} \right\rangle ,$$

leads to a decomposition between production and transport:

$$\mathbf{F}^{(k+1)} \in \arg\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \mid \mathbf{Y}\right), \mathbf{F} \right\rangle , \qquad \text{Production}$$

$$\mathbf{Q}^{(k+1)} \in \arg\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \mid \mathbf{Y}\right), A\mathbf{Q} \right\rangle , \qquad \text{Transport}$$

$$\mathbb{E}\left(\boldsymbol{\lambda}^{(k+1)} \mid \mathbf{Y}\right) = \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \mid \mathbf{Y}\right) + \rho \, \mathbb{E}\left(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)} \mid \mathbf{Y}\right) . \qquad \text{Update}$$

Decomposing the transport problem

The transport subproblem

$$\min_{\mathbf{Q}} \; J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \;\middle|\; \mathbf{Y}\right), A\mathbf{Q} \right\rangle ,$$

writes in a detailled manner

$$\min_{\mathbf{Q}} \sum_{t=0}^{T-1} \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) + \left\langle A^\top \mathbb{E}\left(\boldsymbol{\lambda}_t^{(k)} \;\middle|\; \mathbf{Y}_t\right), \mathbf{Q}_t \right\rangle \Big) .$$

This minimization subproblem is evidently decomposable in time (t by t) and in space (arc by arc), leading to a collection of easy to solve subproblems.

Decomposing the production problem

The production subproblem

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \;\middle|\; \mathbf{Y}\right), \mathbf{F} \right\rangle ,$$

evidently decomposes node by node

$$\min_{\mathbf{F}^i} J^i_{\mathfrak{P}}[\mathbf{F}^i] + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{i,(k)} \;\middle|\; \mathbf{Y}\right), \mathbf{F}^i \right\rangle ,$$

hence a stochastic optimal control subproblem for each node i:

$$\begin{aligned}
\min_{\mathbf{X}^i,\mathbf{U}^i,\mathbf{F}^i} \; & \mathbb{E}\left(\sum_{t=0}^{T-1} \left(L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}^{i,(k)}_t \;\middle|\; \mathbf{Y}_t\right), \mathbf{F}^i_t \right\rangle \right) + K^i(\mathbf{X}^i_T) \right) \\
\text{s.t.} \; & \mathbf{X}^i_{t+1} = f^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) \\
& \mathbf{U}^i_t \preceq \mathcal{F}_t \; .
\end{aligned}$$

Solving the production subproblems by DP

Assuming that

- the process $\mathbf{W}$ is a white noise,
- the process $\mathbf{Y}$ follows a dynamics $\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$,

Dynamic Programming applies for production subproblems:

$$
\begin{aligned}
V_T^i(x,y) &= K^i(x) \\
V_t(x,y) &= \min_{u,f} \mathbb{E}\Big(L_t^i(x,u,f,\mathbf{W}_{t+1}) \\
&\qquad + \left\langle \mathbb{E}\left(\boldsymbol{\lambda}_t^{i,(k)} \mid \mathbf{Y}_t = y\right), f \right\rangle + V_{t+1}^i(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1})\Big) \\
&\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(x,u,f,\mathbf{W}_{t+1}), \\
&\qquad\; \mathbf{Y}_{t+1} = h_t(y, \mathbf{W}_{t+1}).
\end{aligned}
$$

Numerical implementation

Our stack is deeply rooted in Julia language

- Modeling Language: `JuMP`
- Open-source SDDP Solver: `StochDynamicProgramming.jl`
- LP/QP Solver: `Gurobi 7.02`

https://github.com/JuliaOpt/StochDynamicProgramming.jl

Implementation of SDDP and DADP

- Implementing SDDP is straightforward

- DADP implementation is more elaborated:

$$\mathbb{E}\left(\boldsymbol{\lambda}^{(k+1)} \mid \mathbf{Y}\right) = \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \mid \mathbf{Y}\right) + \rho\, \mathbb{E}\left(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)} \mid \mathbf{Y}\right) .$$

  We use a crude relaxation:

  - We choose $\mathbf{Y} = 0$. We denote $\underline{\lambda}^{(k)} = \mathbb{E}\left(\boldsymbol{\lambda}^{(k)}\right)$. The update becomes

$$\underline{\lambda}^{(k+1)} = \underline{\lambda}^{(k)} + \underbrace{\rho}_{\text{Update step}} \underbrace{\mathbb{E}\left(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)}\right)}_{\text{Monte Carlo}} .$$

  - Unfortunately, we do not know the Lipschitz constant of the derivative!
  - And the problem is not even strongly convex ...

We compare three algorithms for gradient ascent

- Quasi-Newton (BFGS): To ensure strong convexity, we add a quadratic term to the cost: $\hat{L}_t^i(.) = L_t^i(.) + u^\top Qu$, with $Q \succ 0$. The update is:

$$\underline{\lambda}^{(k+1)} = \underline{\lambda}^{(k)} + \rho^{(k)} \, \hat{\mathbb{E}}\big\{A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)}\big\} \; .$$

- Alternating Direction Method of Multipliers (ADMM): we add an augmented Lagrangian to solve the problem. The update becomes

$$\underline{\lambda}^{(k+1)} = \underline{\lambda}^{(k)} + \frac{\tau}{2} \, \hat{\mathbb{E}}\big(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)}\big) \; .$$

- Stochastic Gradient Descent (SGD):

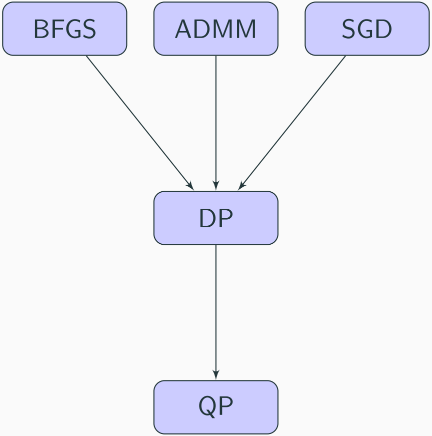
$$\underline{\lambda}^{(k+1)} = \underline{\lambda}^{(k)} + \frac{1}{1+k} \, \big(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)}\big)(\omega) \; .$$

	BFGS	ADMM	SGD
ρ	line search	$\rho^{(k)} \to \tau$	$1/(1+k)$
MC size	100-1000	100-1000	1
software	L-BFGS-B[3]	self	self

[3] The famous implementation of [Zhu et al, 1997]

Double, double toil and trouble

Digesting the stochastic caldron, between time and space ...



- Global problem P

$$\min_{\mathbf{Q},\mathbf{F}} \quad J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}]$$
$$\text{s.t.} \quad A\mathbf{Q} + \mathbf{F} = 0 \ .$$

- Decomposed subproblem P_i

$$J_{\mathfrak{P}}(\mathbf{F}^i) = \min_{\mathbf{X}^i,\mathbf{U}^i,\mathbf{F}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} \big(L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) + \langle \mathbb{E}(\boldsymbol{\lambda}_t^{i,(k)} \mid \mathbf{Y}_t), \mathbf{F}_t^i \rangle \big) + K^i(\mathbf{X}_T^i) \Big)$$
$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1})$$

- DP subproblem V_t^i

$$V_t^i(x, y) = \min_{u,f} \mathbb{E}\Big(L_t^i(x, u, f, \mathbf{W}_{t+1}) + \langle \mathbb{E}(\boldsymbol{\lambda}_t^{i,(k)} \mid \mathbf{Y}_t = y), f \rangle + V_{t+1}^i(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1}) \Big)$$

Results — Monthly

Compute Bellman value functions at monthly time steps ($T = 12$).

$\mathfrak{n}_{bin}$	1	2	5
SDDP value	5.048	5.203	$+\infty$
SDDP time	0.5"	87"	$+\infty$
BFGS value	5.088	5.202	5.286
BFGS time	18"	49"	161"
ADMM value	5.087	5.201	5.288
ADMM time	14"	49"	66"
SGD value	5.088	5.202	5.292
SGD time	37"	66"	130"

- SDDP does not converge if $\mathfrak{n}_{bin} = 5$.
- If $\mathfrak{n}_{bin} = 1$, SDDP is better than DADP because of the discretization scheme used in Dynamic Programming.
- BFGS and ADMM compute a gradient with Monte-Carlo ...
- BFGS does not solve the original problem (strong convexification)

Results — Weekly

Compute Bellman value functions at weekly time steps ($T = 52$).

$\mathfrak{n}_{bin}$	1	2	5
SDDP value	9.396	9.687	$+\infty$
SDDP time	8''	928''	$+\infty$
BFGS value	9.411	9.687	9.974
BFGS time	69''	157''	575''
ADMM value	9.404	9.682	9.984
ADMM time	65''	326''	643''
SGD value	9.411	9.679	9.971
SGD time	194''	281''	712''

- The longer the horizon, the slower SDDP is.
- Here, BFGS is penalized by line-search, as it uses an approximated gradient
- SGD works quite well compared to BFGS and ADMM: these two algorithms are penalized by the Monte-Carlo computation of the gradient.

Multipliers convergence

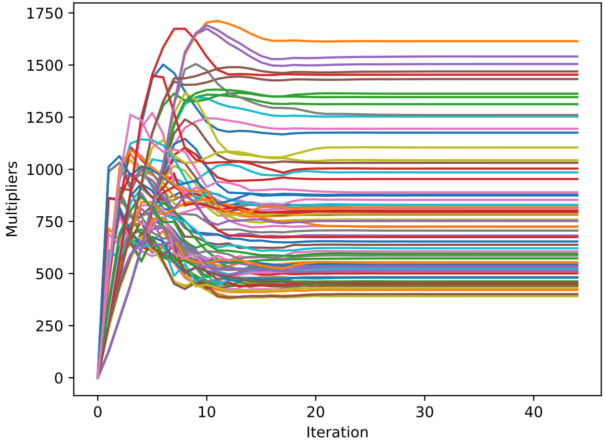


Figure 1: Convergence of multipliers with BFGS ($T = 12$, $\mathfrak{n}_{bin} = 1$).

SDDP convergence

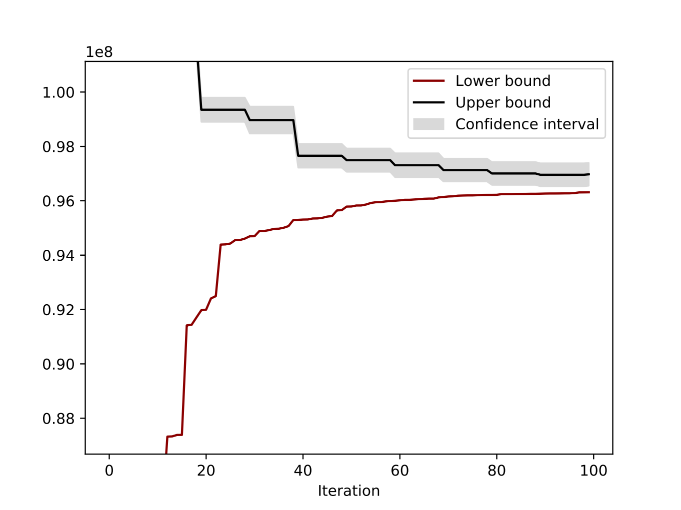


Figure 2: Convergence of SDDP's upper and lower bounds ($T = 52$, $\mathfrak{n}_{bin} = 2$).

Conclusion

Conclusion

Conclusion

- A survey of different algorithms, mixing spatial and time decomposition.
- DADP works well with the crude relaxation $\mathbf{Y} = 0$, and even beats SDDP if $\mathfrak{n}_{bin} \geq 2$.
- We had a lot of troubles to deal with approximate gradients!

Perspectives

- Find a proper information process $\mathbf{Y}$.
- Improve the integration between SDDP and DADP.
- Test other decomposition schemes (by quantity, by prediction).

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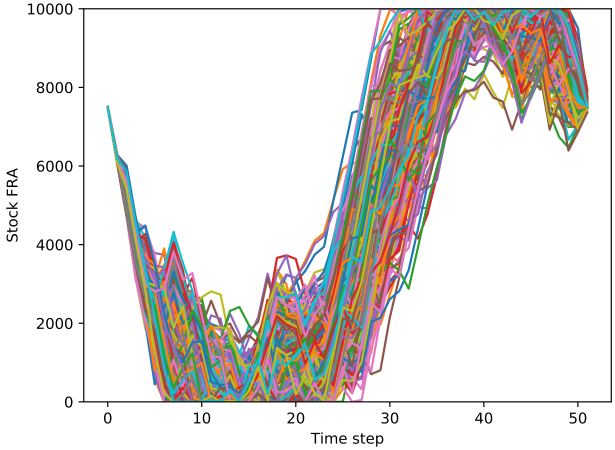
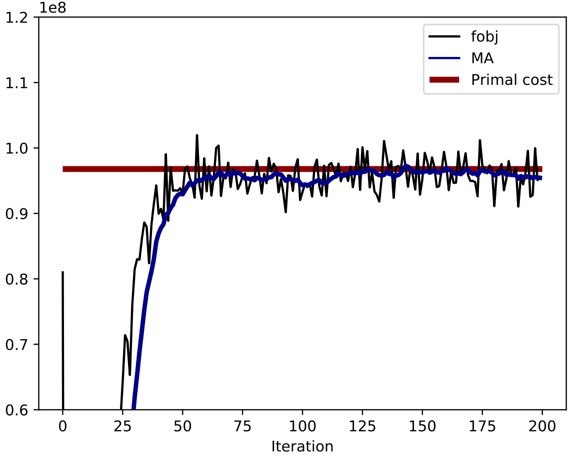
Dams trajectory



SGD convergence

Plotting the convergence with $T = 52$ and $\mathfrak{n}_{bin} = 2$.



ADMM convergence

Plotting the logarithm of the norm of the primal residual with $T = 52$ and $\mathfrak{n}_{bin} = 5$.