Optimal Control of a Domestic Microgrid

F. Pacaud, R. Bonabe de Rougé
Advisors: P. Carpentier, J.-P. Chancelier, M. De Lara

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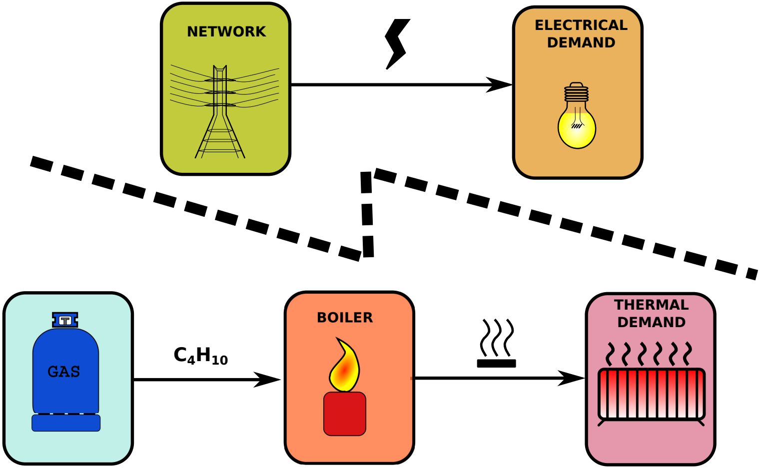
efficacity

A partnership between mathematicians and thermicians

- Efficacity is a research institute for energy transition — an original mix of companies and academic researchers
- This presentation sums up a common work between Cermics and Efficacity
- This cooperation aims to apply optimization algorithms to real world problems

In a “classical” energy system, thermal and electrical energy management are usually treated apart



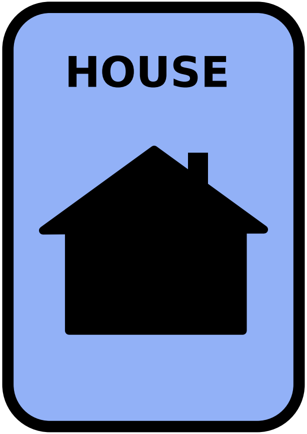
Is it worth to equip the system with
a combined heat and power generator (CHP)
together with a battery?

Challenges:

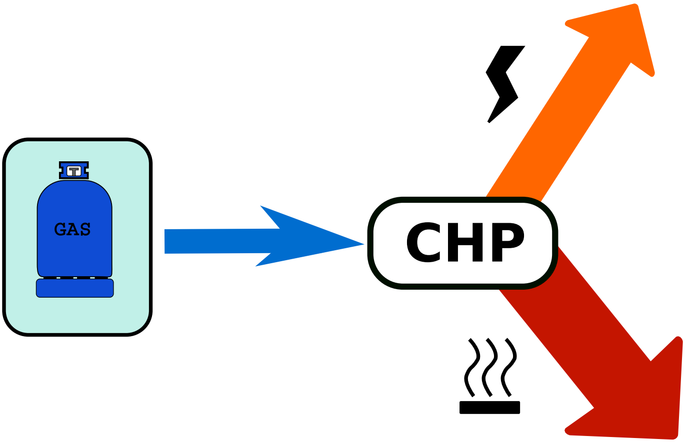
- CHP is either ON or OFF, and always produces the same amount of electricity and heat
- Thermal and electrical system are coupled with the CHP
- Two storages devices (battery and hot water tank) with a dynamic
- Prices and setpoints vary along time, rendering necessary the two storages

We turn to mathematical optimization to answer the question

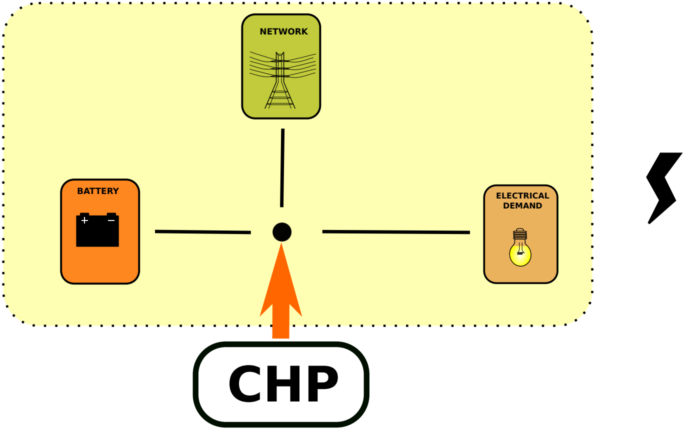
Our system



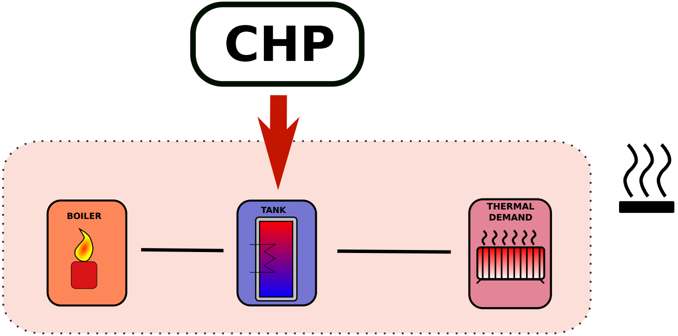
What is a Combined Heat and Power Generator?



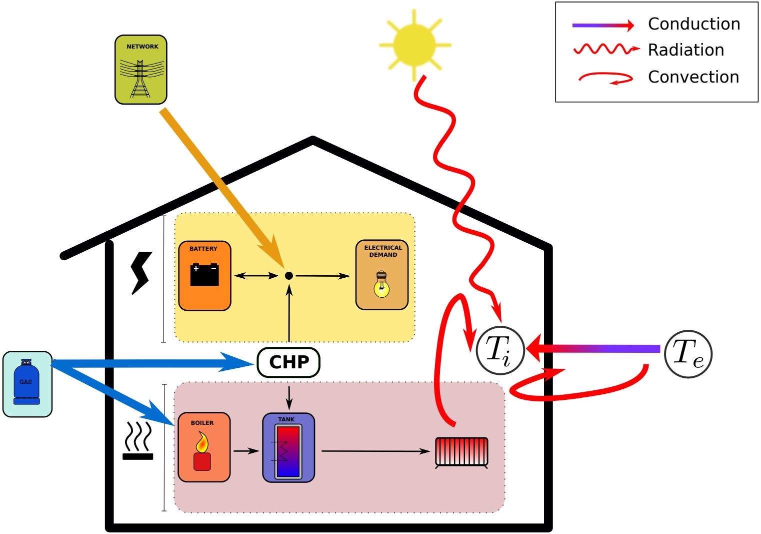
What electrical system are we considering?



What thermal system are we considering?



Problem's description



What do we aim to do?

We want to:

- Minimize gas' consumption
- Minimize electricity imported from the network
- Maintain a comfortable temperature inside the house

To achieve these goals, we can:

- Switch on/off the CHP
- Store electricity in battery and heat in hot water tank
- Control auxiliary boiler and heaters' inflow

We consider **15mn** timesteps

Outline

1. Mathematical formulation
   - Model
   - Randomness
   - Optimization problem
2. Numerical resolution
   - Methods
   - Assessment
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3. Conclusion

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We introduce states, controls and noises

- **Stock variables** $X_t = \big(B_t, H_t, \theta^i_t, \theta^w_t\big)$
  - B_t, battery level (kWh)
  - H_t, hot water storage (kWh)
  - θ^i_t, inner temperature ($^\circ C$)
  - θ^w_t, wall's temperature ($^\circ C$)
- **Control variables** $U_t = \big(Y_t, F_{B,t}, F_{A,t}, F_{H,t}\big)$
  - $Y_t \in \{0, 1\}$ boolean ON/OFF CHP generator control variable
  - $F_{B,t}$, energy stored in the battery
  - $F_{A,t}$, energy produced by the auxiliary boiler
  - $F_{H,t}$, thermal heating
- **Perturbations** $W_t = \big(D^E_t, N_t, P^{ext}_t, \theta^e_t\big)$
  - D^E_t, electrical demand (kW)
  - N_t, occupancy (integer)
  - P^{ext}_t, external radiations (kW)
  - θ^e_t, external temperature ($^\circ C$)

Discrete time state equations

So we have the four state equations (all linear):

$$B_{t+1} = \alpha_B B_t - \beta_B F_{B,t}$$

$$H_{t+1} = \alpha_H H_t + \beta_H \big[F_{A,t} + F_{GH,t} - F_{H,t} \big]$$

$$\theta^w_{t+1} = \theta^w_t + \frac{\Delta T}{c_m} \left[\frac{\theta^i_t - \theta^w_t}{R_i + R_s} + \frac{\theta^e_t - \theta^w_t}{R_m + R_e} + \gamma F_{H,t} + \frac{R_i}{R_i + R_s} P^{int}_t + \frac{R_e}{R_e + R_m} P^{ext}_t \right]$$

$$\theta^i_{t+1} = \theta^i_t + \frac{\Delta T}{c_i} \left[\frac{\theta^w_t - \theta^i_t}{R_i + R_s} + \frac{\theta^e_t - \theta^i_t}{R_v} + \frac{\theta^e_t - \theta^i_t}{R_f} + (1-\gamma) F_{H,t} + \frac{R_s}{R_i + R_s} P^{int}_t + P_{occ} \right]$$

which will be denoted:

$$\boxed{X_{t+1} = f_t(X_t, U_t, W_{t+1})}$$

Optimization criterion

- Cost to import electricity from the network

$$-\underbrace{b_E \max\{0, -F_{NE,t+1}\}}_{\text{selling}} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{\text{buying}}$$

where we define the recourse variable (electricity balance):

$$\underbrace{F_{NE,t+1}}_{Network} = \underbrace{D^E_{t+1}}_{Demand} - \underbrace{F_{B,t}}_{Battery} - \underbrace{P^E_{chp} \times Y_t}_{CHP}$$

- Cost to use the CHP

$$\pi_{chp} \times Y_t$$

- Cost to use auxiliary burner

$$\pi_{gas} \times F_{A,t}$$

- Virtual Cost of thermal discomfort

$$\kappa_{th}(\theta^i_t - \bar{\theta}^i_t)$$

We penalize the discrepancy between the indoor temperature θ^i_t and a setpoint $\bar{\theta}^i_t$ with a piecewise linear cost κ_{th}

Instantaneous and final costs

- The instantaneous convex costs are

$$\mathcal{C}_t(X_t, U_t, W_{t+1}) = \underbrace{\pi_{chp} Y_t}_{CHP} + \underbrace{\pi_{gas} F_{A,t}}_{Aux.\ Burner} \\ \underbrace{- b_E \max\{0, -F_{NE,t+1}\}}_{buying} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{selling} \\ + \underbrace{\kappa_{th}(\theta^i_t - \bar{\theta}^i_t)}_{discomfort}$$

- We add a final linear cost

$$-\pi_H H_{T_f} - \pi_B B_{T_f}$$

to avoid empty stocks at the final horizon T_f

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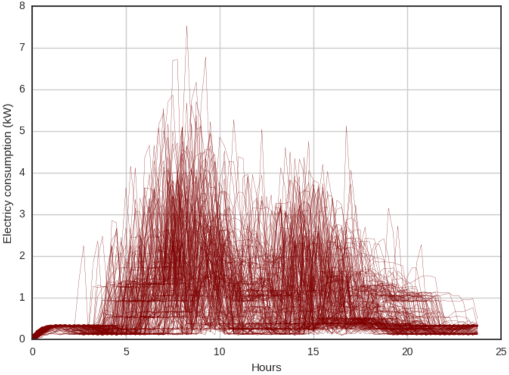
Those perturbations are highly variable

Different scenarios for electrical demand:



We model perturbations as random variables

- We recall that $W_t = \left(D_t^E, N_t, P_t^{ext}, \theta_t^e\right)$ with:
  - D_t^E, electrical demand (kW)
  - N_t, occupancy (integer)
  - P_t^{ext}, external radiations (kW)
  - θ_t^e, external temperature ($^\circ C$)
- We model W_t as random variables upon $\left(\Omega, \mathcal{A}, \mathbb{P}\right)$

$$W_t : \Omega \to \mathbb{R}^4$$

  so that $(W_1, \dots, , W_{T_f})$ forms a stochastic process
- We recall that W_{t+1} stand for the exogeneous perturbations during the time interval $[t, t+1[$

We need to add the nonanticipativity constraints

- σ-algebra

$$\mathcal{A}_t = \sigma(W_1, \ldots, W_t)$$

- Non-anticipativity constraint

$$U_t = \left(Y_t, F_{B,t}, F_{A,t}, F_{H,t}\right)$$
$$\text{is } \mathcal{A}_t\text{–measurable}$$

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That gives the following stochastic optimization problem

$$
\begin{aligned}
\min_{X,U} \quad & \mathbb{E}\left[\sum_{t=0}^{T_f-1} \underbrace{\mathcal{C}(X_t, U_t, W_{t+1})}_{instantaneous \ \ cost} \underbrace{-\pi_H H_{T_f} - \pi_B B_{T_f}}_{final \ \ cost} \right] \\
s.t \quad & X_{t+1} = f(X_t, U_t, W_{t+1}) \\
& B^\flat \leq B_t \leq B^\sharp \\
& H^\flat \leq H_t \leq H^\sharp \\
& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
& F_i^\flat \leq F_{i,t} \leq F_i^\sharp \ , \ \ \forall i \in \{B, A, H\} \\
& U_t \preceq \mathcal{A}_t
\end{aligned}
$$

That gives the following stochastic optimization problem

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\min_{X,U} \quad & \mathbb{E}\left[\sum_{t=0}^{T_f-1} \underbrace{\mathcal{C}(X_t, U_t, W_{t+1})}_{instantaneous\ \ cost} \underbrace{-\pi_H H_{T_f} - \pi_B B_{T_f}}_{final\ \ cost}\right] \\
s.t \quad & X_{t+1} = f(X_t, U_t, W_{t+1}) \qquad \text{Dynamic} \\
& B^\flat \leq B_t \leq B^\sharp \\
& H^\flat \leq H_t \leq H^\sharp \\
& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
& F_i^\flat \leq F_{i,t} \leq F_i^\sharp \ , \ \ \forall i \in \{B, A, H\} \\
& U_t \preceq \mathcal{A}_t \qquad \text{Measurability}
\end{aligned}
$$

Where are we now? And where are we heading to?

- The problem is formulated
- Now, we are going to present two methods to tackle this problem
- Does it pays to equip the system with a CHP?

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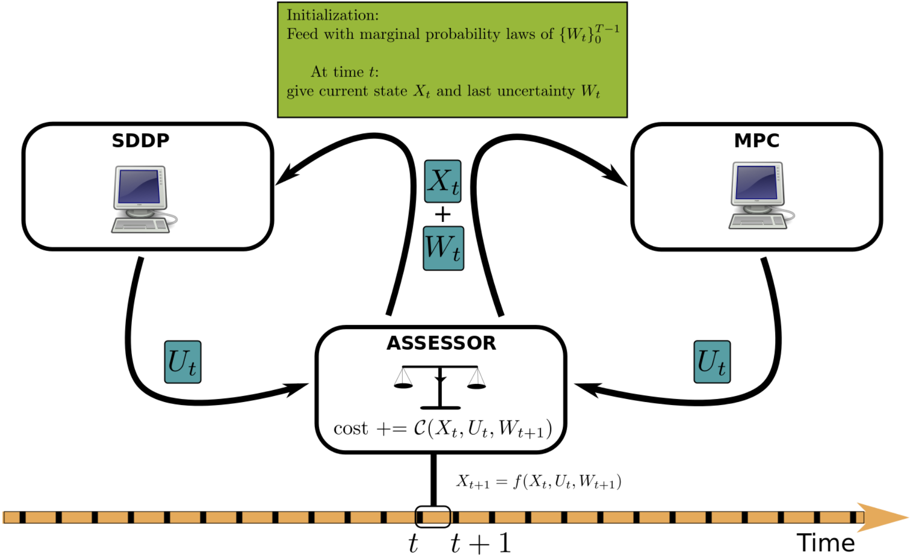
We are going to compare two methods

MPC
Model Predictive Control

SDDP
Stochastic Dual Dynamic Programming

How are we going to evaluate these two methods?



Model Predictive Control

At the beginning of time period $[\tau, \tau+1]$, do

- Consider a **rolling horizon** $[\tau, \tau+H[$
- Consider a **deterministic scenario** of demands (forecast) $\left(\overline{W}_{\tau+1}, \dots, \overline{W}_{\tau+H}\right)$
- Solve the **deterministic optimization** problem over this rolling horizon to get optimal solution $(U_\tau, \dots, U_{\tau+H})$ over horizon $H = 24h$

$$
\begin{aligned}
\min_{X,U} \; & \left[\sum_{t=\boldsymbol{\tau}}^{\boldsymbol{\tau+H}} \mathcal{C}(X_t, U_t, \overline{W}_{t+1}) - \pi_H H_{T_f} - \pi_B B_{T_f} \right] \\
s.t. \quad & X_\cdot = (X_{\boldsymbol{\tau}}, \dots, X_{\tau+H}), \quad U_\cdot = (U_{\boldsymbol{\tau}}, \dots, U_{\tau+H-1}) \\
& X_{t+1} = f(X_t, U_t, \overline{\mathbf{W}}_{\mathbf{t+1}}) \\
& B^\flat \leq B_t \leq B^\sharp \\
& H^\flat \leq H_t \leq H^\sharp \\
& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
& F_{i,t} \leq F_i^\sharp \ , \quad \forall i \in \{B, A, H\}
\end{aligned}
$$

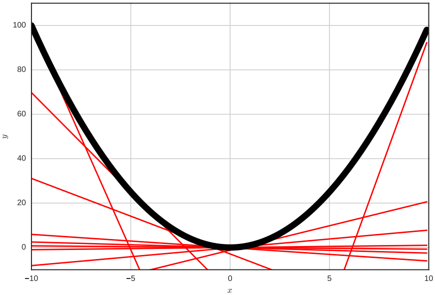
- Use only control U_τ, and iterate at time $\tau + 1$

Stochastic Dual Dynamic Programming

Dynamic Programming

Use marginal laws μ_t of uncertainties to estimate expectation and compute **offline** value functions with the backward equation:

$$V_t(x_t) = \min_{U_t} \mathbb{E}_{\mu_t}\Big[\underbrace{\mathcal{C}_t(x_t, U_t, W_{t+1})}_{\text{current cost}} + \underbrace{V_{t+1}\Big(f(x_t, U_t, W_{t+1})\Big)}_{\text{future costs}}\Big]$$



SDDP

Convex value functions V are approximated as a supremum of a finite set of affine functions

Online computation

At the beginning of $[\tau, \tau+1[$, the assessor sends the current state X_τ and w_τ

MPC

- Set $\overline{W}_{\tau+1} = w_\tau$,
  $\overline{W}_{\tau+i} = \mathbb{E}\big(W_{t+i}\big) \qquad \forall i \geq 2$
- Consider as forecast
  $(\overline{W}_{\tau+1}, \overline{W}_{\tau+2}, \dots, \overline{W}_T)$
- Solve optimization problem
- Send $U^\#_\tau$ to assessor

SDDP

- Having computed $\big(\widetilde{V}_t\big)_0^{T_f}$, solve:

$$U^\#_\tau = \arg\min_{U_\tau} \Big[\mathcal{C}_t(X_\tau, U_\tau, w_\tau) + \widetilde{V}_{\tau+1}\Big(f_\tau(X_\tau, U_\tau, w_\tau)\Big)\Big]$$

- Send $U^\#_\tau$ to assessor

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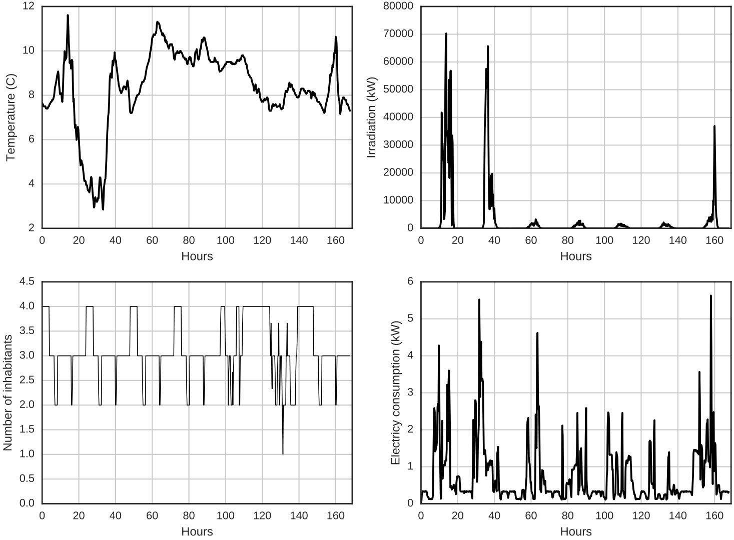
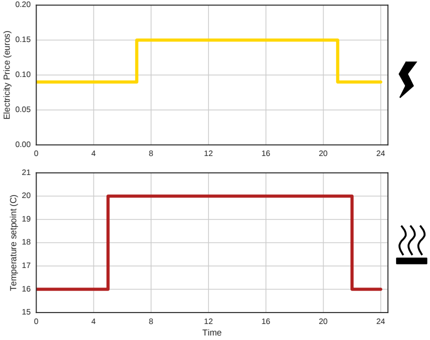
How to assess management methods?

We consider one week in winter and 200 assessment scenarios



We define settings for our problem



- $T_f = 24\text{h}$, $\Delta T = 15\text{mn}$
- Electricity peak and off-peak hours
  - $\pi_{elec} = 0.09$ or 0.15 euros/kWh
  - $\pi_{gas} = 0.06$ euros/kWh
- Temperature set-point
  $16^\circ C$ or $20^\circ C$
- Empty stocks at midnight

$$\pi_H = 0\,, \quad \pi_B = 0$$

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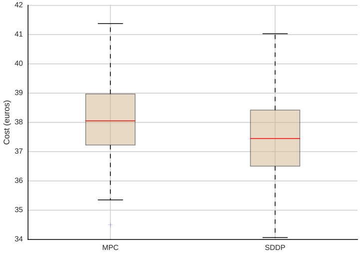
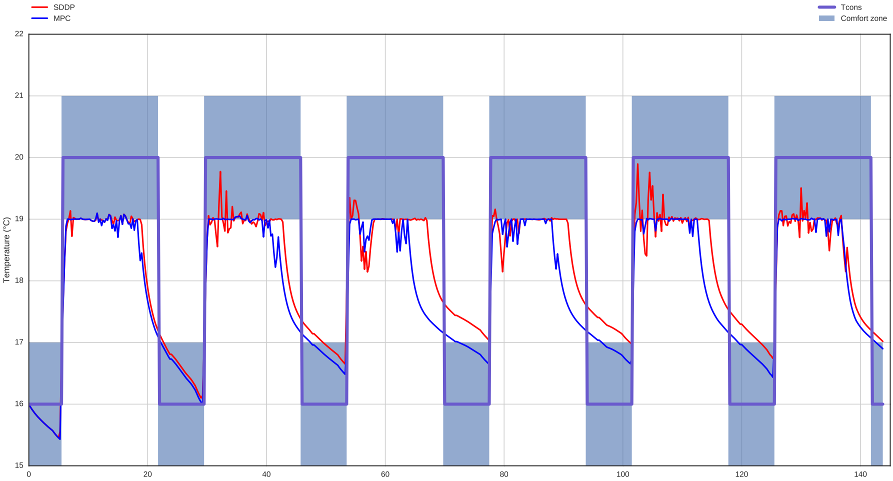
Now, we assess the two methods by their costs over assessment scenario



	euros/week	%
no CHP, no battery	46.84	ref
MPC	38.08	- 18.7%
SDDP	37.46	- 20.1 %

Evolution of temperatures during the week



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Conclusion

- We gain around 7 euros per week to equip the system with a CHP (not that much, but we consider only one house)
- It pays to control the system with SDDP over the considered scenario
- With SDDP, offline computations take some time (15mn) but online computations are straightforward
- We could use more online information (updated forecast)
- Those results could be used to make an economic evaluation

Perspectives

Use decomposition/coordination algorithms to control an urban neighbourhood