Solving a continent-scale, multi-attribute inventory routing problem at Renault

One-day workshop on the IRP

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Renault - Ecole des Ponts

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Introduction

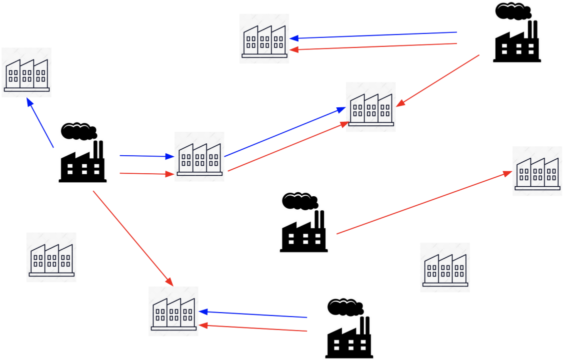

Figure 1: Illustration of the Inventory Routing Problem (only one day represented).

Europe instance

Figure 2: Visual of the Europe instance.

Depots, customers and routes

Sites: Multi-depot instances, multicommodity release and demand.

Vehicles: Infinite fleet of homogeneous vehicles.

Routes:

- Transport may last several days as considered by Lagos, Boland, and Savelsbergh [11].
- Vehicles have size L: a 1D bin packing problem is solved.
- Split deliveries are allowed.
- Limit on the number of stops $S_{\max}$ per route.

Our optimization problem

$$\min_{\text{routes, inventory}} \left| \begin{array}{l} \quad \text{routes' cost} \\ + \text{depots' inventory cost} \\ + \text{customers' inventory cost} \\ + \text{customers' shortage cost} \end{array} \right. \qquad \text{(IRP)}$$

$$\text{s.t.} \left| \begin{array}{l} \text{Routes are feasible.} \\ \text{Inventory dynamics are respected.} \\ \text{Inventory dynamics and routes are coherent.} \end{array} \right.$$

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Existing approaches

We can distinguish several trends:

- Branch and bound Coelho and Laporte [6] and Desaulniers, Rakke, and Coelho [9].
- Metaheuristics Benoist et al. [3].
- Matheuristics Archetti, Boland, and Speranza [1], Bertazzi et al. [4], Su et al. [13], Coelho, De Maio, and Laganà [7], and Archetti et al. [2].
- Two-step heuristics Campbell and Savelsbergh [5] and Cordeau et al. [8].

Challenges

The main difficulties we face are:

- The multi-depot and multi-commodity aspects.
- The multiple-day routes.
- The size of the instances: $|D| = 15$, $|C| = 600$, $|M| = 30$, $T = 20$ compared with the benchmark *OR-Brescia - Benchmark Instances* [12].

$\rightarrow$ No algorithm is known to scale to our context.

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Overview

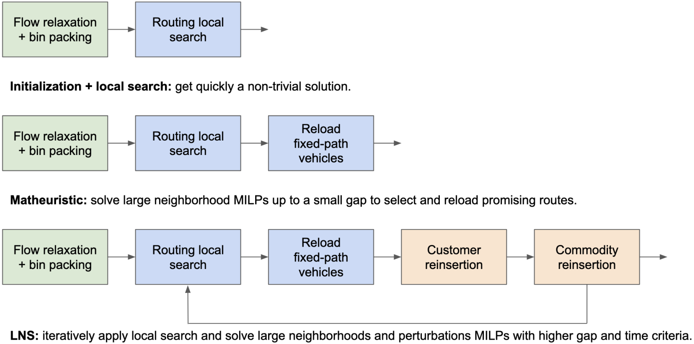

Figure 3: Algorithms principles.

Contributions

- Review the literature.
- Implement and adapt 13 route neighborhoods.
- Adapt a matheuristic as large neighborhood.
- Design two new perturbations.

$\rightarrow$ Scale to our instances.

Reload fixed-path vehicles principle

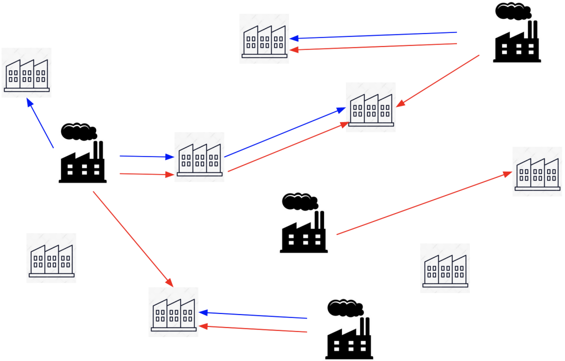

Figure 4: Initial solution (only one day represented).

Reload fixed-path vehicles principle

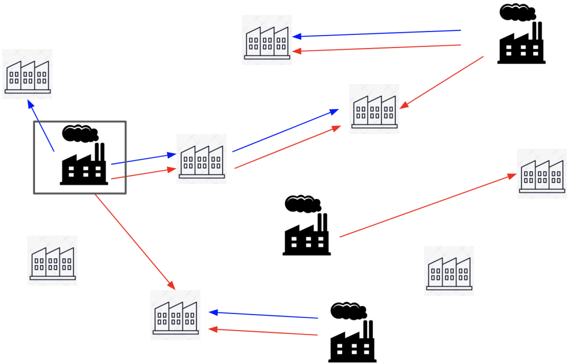

Figure 5: Routes related to one depot.

Reload fixed-path vehicles principle

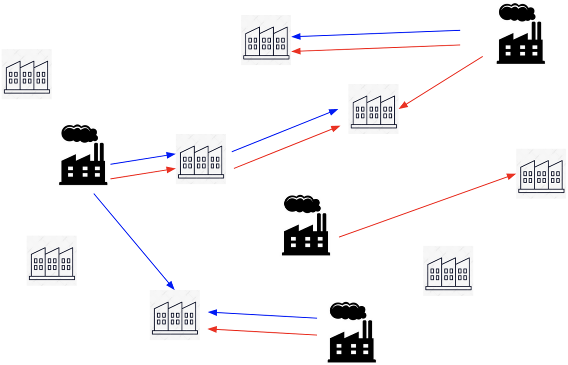

Figure 6: New solution.

Reload fixed-path vehicles

Ideas:

- Neighborhood version of Bertazzi et al. [4] and Archetti, Boland, and Speranza [1]
- Simultaneously choose the routes to keep and the quantities to be delivered.

Remark: this neighborhood can be applied on any set of routes.

Implementation: We use Gurobi solver Gurobi Optimization, LLC [10] with a warm start $\rightarrow$ a few seconds of computation per depot.

Customer reinsertion principle

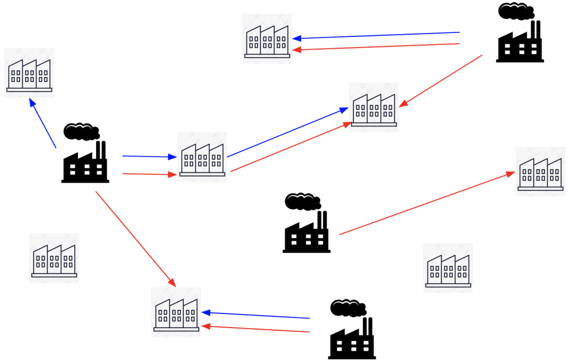

Figure 7: Initial solution (only one day represented).

Customer reinsertion principle

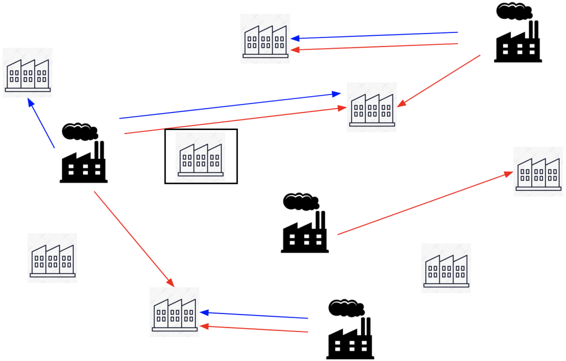

Figure 8: Removal of a customer (only one day represented).

Customer reinsertion principle

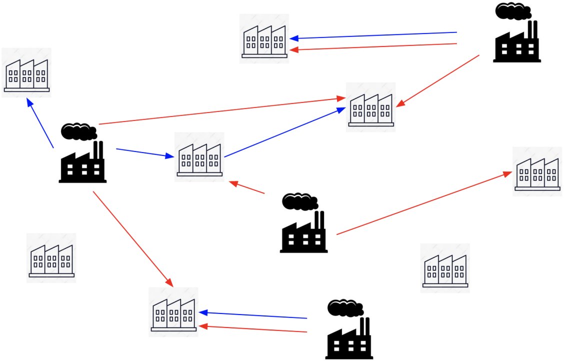

Figure 9: Reinsertion of a customer (only one day represented).

Customer reinsertion

Ideas:

- Use coupled flows.
- Pre-compute as much cost as we can.
- Sparsify the graphs.

Proposition

Our MILP is a relaxation of the customer reinsertion problem.

- **Remark**: We systematically reconstruct a feasible solution.
- **Implementation**: We use Gurobi solver Gurobi Optimization, LLC [10] with a warm start $\rightarrow$ a few seconds of computation per customer.

Commodity reinsertion principle

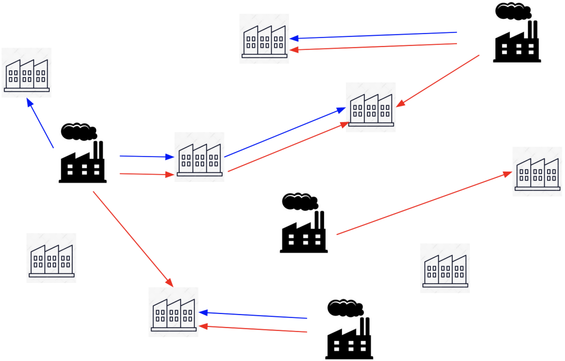

Figure 10: Initial solution (only one day represented).

Commodity reinsertion principle

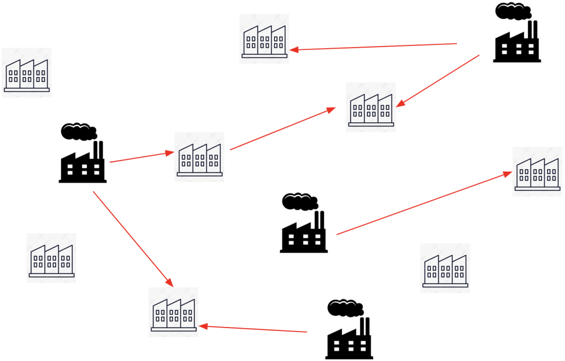

Figure 11: Removal of a commodity (only one day represented).

Commodity reinsertion principle

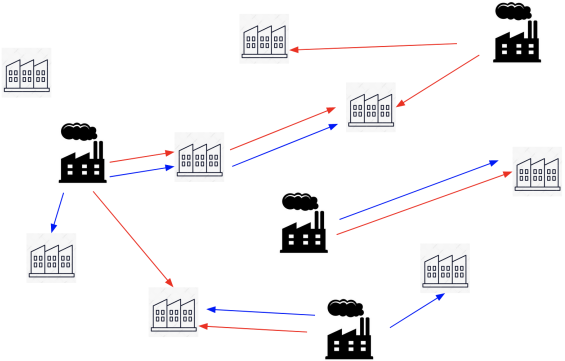

Figure 12: Reinsertion of a commodity (only one day represented).

Commodity reinsertion

Ideas:

- Use 2 coupled flows.
- Restrict to new direct routes.
- Combine with a fast local search.

Remark: We systematically reconstruct a feasible solution.

Implementation: We use Gurobi solver Gurobi Optimization, LLC [10] and warm start $\rightarrow$ a few seconds of computation per commodity.

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Global results of the LNS

Number of instances	71
Average number of depots	15
Average number of customers	602
Average number of commodities	30
Average horizon (days)	21

Table 1: Instances overview.

	$S_{\max} = 3$	$S_{\max} = 10$
Gap initialization + local search	121%	77%
Gap matheuristic	87%	64%
Gap LNS	66%	40%
Time limit	90 minutes	180 minutes

Table 2: Average gap results.

Time per operator

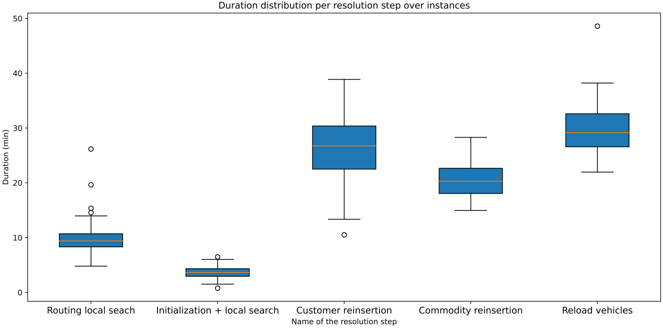

Figure 13: Duration allocated per operator.

Cost gain per CPU time and operator

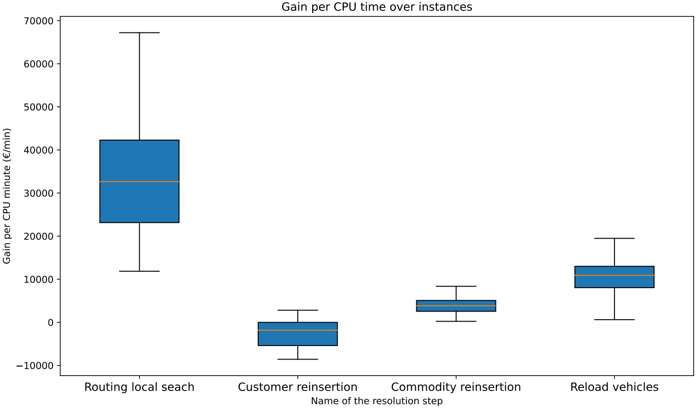

Figure 14: Cost gain per CPU time and operator.

Gaps cumulative distributions

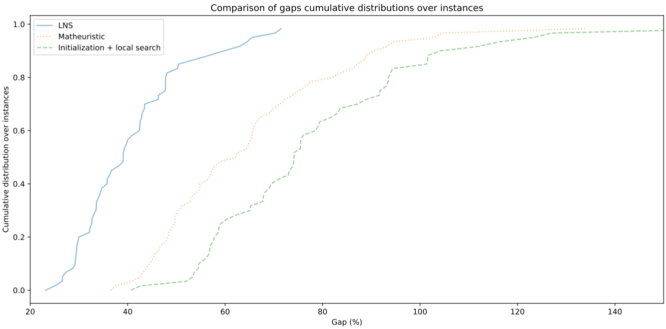

Figure 15: Cumulative distribution of the gaps for $S_{\max} = 10$.

Gaps cumulative distributions after ablation

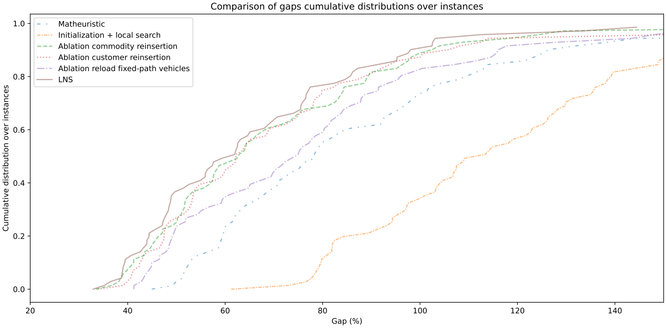

Figure 16: Cumulative distribution of the gap over instances.

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Conclusion

Literature review on IRP.

LNS (to be submitted):

- 13 routing neighborhoods adapted.
- One route-based large neighborhood.
- Two new perturbations.
- Implementation and numerical experiments.

Perspectives:

- Industrialization.
- Address the "forecast dispatch" problem.
- Use machine learning for operations research techniques.

Bibliography I

[1] Claudia Archetti, Natashia Boland, and M. Grazia Speranza. "A Matheuristic for the Multivehicle Inventory Routing Problem". In: *INFORMS Journal on Computing* 29.3 (Apr. 2017), pp. 377–387. ISSN: 1091-9856. DOI: 10/gbsn75.

[2] Claudia Archetti et al. "A Kernel Search Heuristic for the Multivehicle Inventory Routing Problem". In: *International Transactions in Operational Research* 28.6 (Nov. 2021), pp. 2984–3013. ISSN: 0969-6016, 1475-3995. DOI: 10/gkzwqh.

[3] Thierry Benoist et al. "Randomized Local Search for Real-Life Inventory Routing". In: *Transportation Science* 45.3 (Mar. 2011), pp. 381–398. ISSN: 0041-1655. DOI: 10.1287/trsc.1100.0360.

[4] Luca Bertazzi et al. "A Matheuristic Algorithm for the Multi-Depot Inventory Routing Problem". In: *Transportation Research Part E: Logistics and Transportation Review* 122 (Feb. 2019), pp. 524–544. ISSN: 1366-5545. DOI: 10/ggdvkf.

[5] Ann Melissa Campbell and Martin W. P. Savelsbergh. "A Decomposition Approach for the Inventory-Routing Problem". In: *Transportation Science* 38.4 (Nov. 2004), pp. 488–502. ISSN: 0041-1655. DOI: 10/fkn54g.

[6] Leandro C. Coelho and Gilbert Laporte. "The Exact Solution of Several Classes of Inventory-Routing Problems". In: *Computers & Operations Research* 40.2 (Feb. 2013), pp. 558–565. ISSN: 0305-0548. DOI: 10/gjvm3n.

[7] Leandro Callegari Coelho, Annarita De Maio, and Demetrio Laganà. "A Variable MIP Neighborhood Descent for the Multi-Attribute Inventory Routing Problem". In: *Transportation Research Part E: Logistics and Transportation Review* 144 (Dec. 2020), p. 102137. ISSN: 13665545. DOI: 10/gm954p.

Bibliography II

[8] Jean-François Cordeau et al. "A Decomposition-Based Heuristic for the Multiple-Product Inventory-Routing Problem". In: *Computers & Operations Research* 55 (Mar. 2015), pp. 153–166. ISSN: 03050548. DOI: 10/f6w8pp.

[9] Guy Desaulniers, Jørgen G. Rakke, and Leandro C. Coelho. "A Branch-Price-and-Cut Algorithm for the Inventory-Routing Problem". In: *Transportation Science* 50.3 (Oct. 2015), pp. 1060–1076. ISSN: 0041-1655. DOI: 10/f8zpgc.

[10] Gurobi Optimization, LLC. *Gurobi Optimizer Reference Manual*. 2021. URL: https://www.gurobi.com.

[11] Felipe Lagos, Natashia Boland, and Martin Savelsbergh. "The Continuous-Time Inventory-Routing Problem". In: *Transportation Science* (Jan. 2020), trsc.2019.0902. ISSN: 0041-1655, 1526-5447. DOI: 10.1287/trsc.2019.0902.

[12] *OR-Brescia - Benchmark Instances*. https://or-brescia.unibs.it/instances.

[13] Zhouxing Su et al. "A Matheuristic Algorithm for the Inventory Routing Problem". In: *Transportation Science* (Mar. 2020), trsc.2019.0930. ISSN: 0041-1655, 1526-5447. DOI: 10/gm954w.