A Two-Echelon Inventory-Routing Problem

S. Charaf[1], D. Taş[2], S.D. Flapper[1], T. van Woensel[1]

[1]Eindhoven University of Technology, The Netherlands
[2]MEF University, Turkey

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s.charaf@tue.nl

Content

Motivation

- Many applications: city logistics, grocery distribution, e-commerce ...
- Last mile increasingly congested
- Sustainable transport

Literature

Two-Echelon Vehicle Routing Problem

Perboli et al. (2011), Breunig et al. (2016), and Marques et al. (2020)

Inventory-Routing Problem

Archetti et al. (2007), Guimarães et al. (2020), and Desaulniers et al. (2016)

Two-Echelon Inventory-Routing Problem

Rohmer et al. (2019) and Farias et al. (2021)

Problem Description

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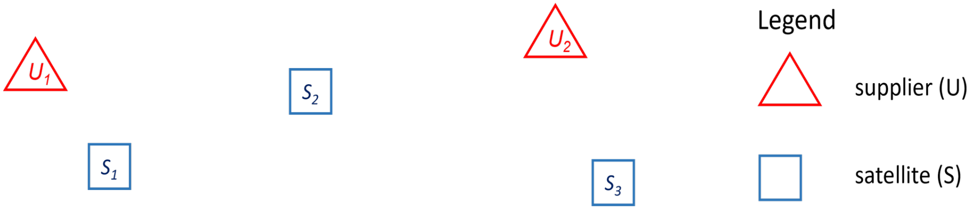

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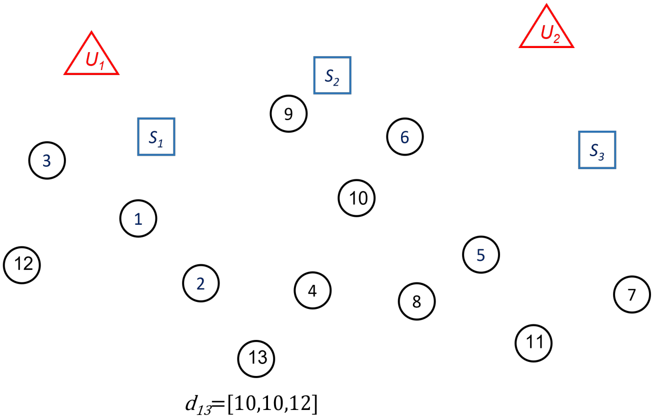


Legend

supplier (U)

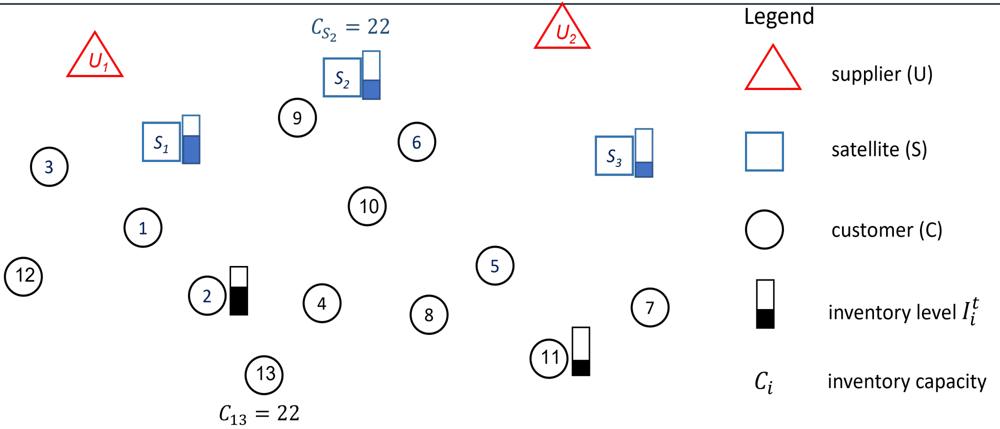
satellite (S)

customer (C)

$T=[1,..,\tau]$ time horizon

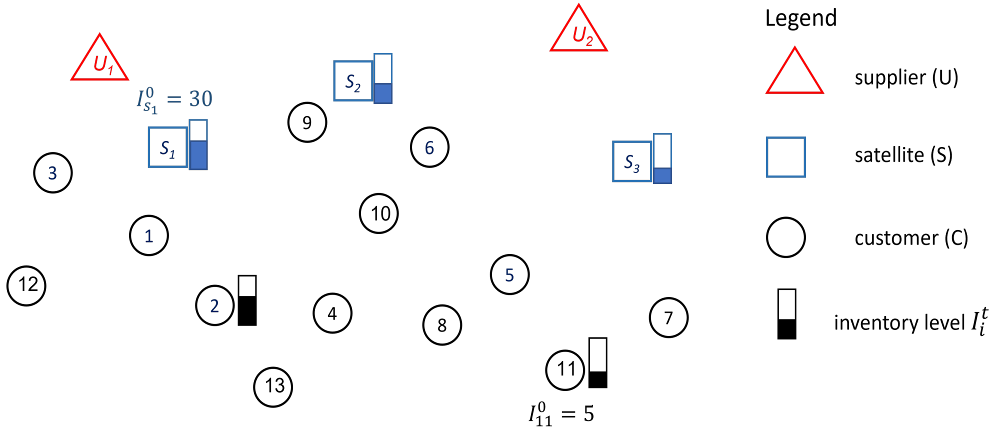
$d_i=[.,.,.,.]$ customer demand

Problem Description



- Vendor Managed Inventory
- Maximum level policy

Problem Description



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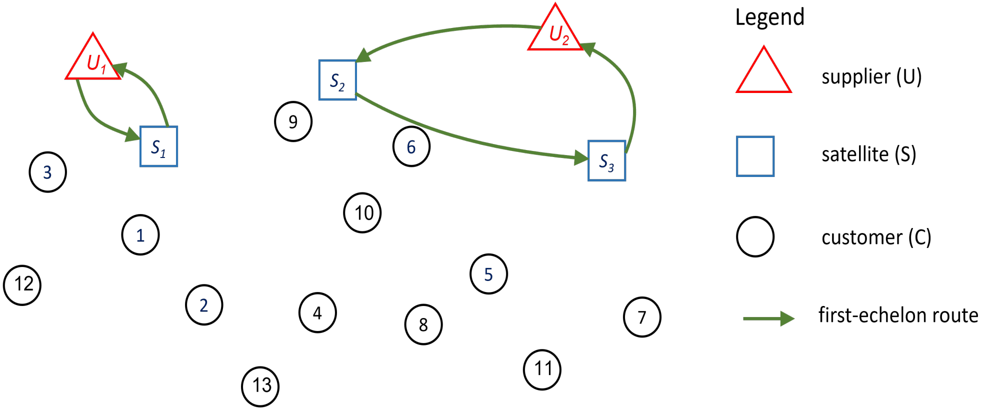

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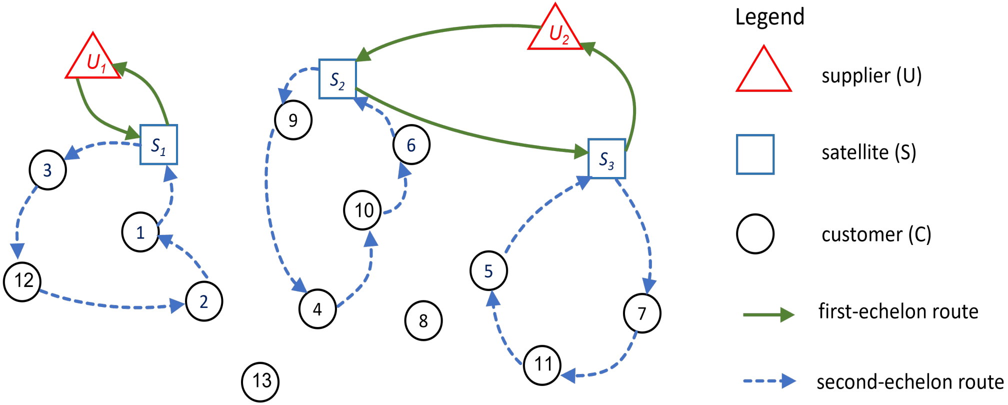
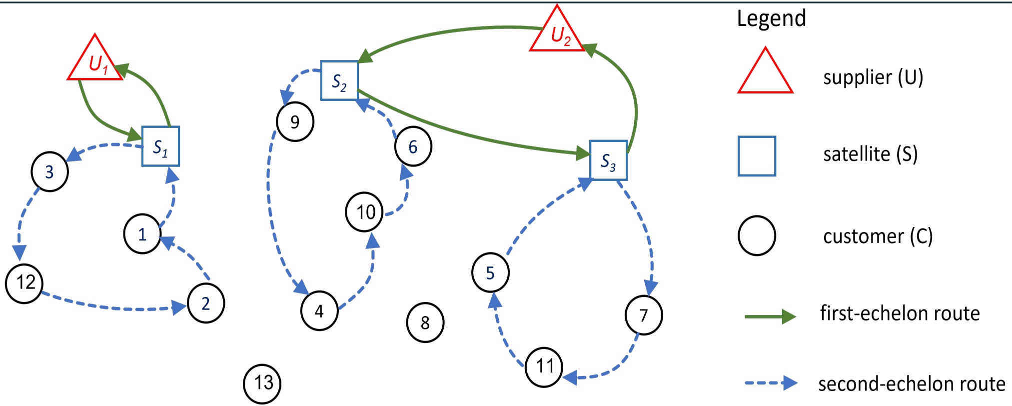


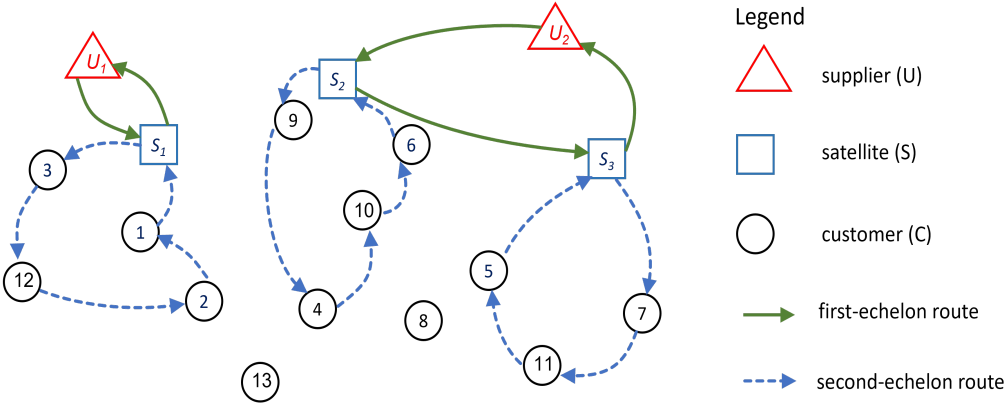
Minimize transportation and inventory costs

Problem Description



Routing constraints:
(1) Vehicle capacity & (2) Nb of vehicles available
(3) Single visit to a customer/satellite

Problem Description



Inventory constraints:

(1) Inventory capacity & (2) no stock-out

Notation and Definitions

First-echelon

- Q^1: first-echelon vehicle capacity
- K^1: number of 1E-vehicles available
- P: set of first-echelon routes

Variables

- $\lambda_p^t \in \{0, 1\}; \ p \in P, \ t \in T$
- $\psi_{s,t}^l \geq 0; \quad s \in S, \ t \in T \cup \{\tau + 1\}, \ l \in \{0\} \cup T$

Notation and Definitions

Example

$\psi^{l}_{s,t} \geq 0; \qquad s \in S,\ t \in T \cup \{\tau + 1\},\ l \in \{0\} \cup T$

$T = [1, 2]$

- $l = 0$: initial = 15
- $l = 1$: qty entering = 30, exiting = 20
- $l = 2$: qty entering = 00, exiting = 20

Notation and Definitions

Example

$\psi^{l}_{s,t} \geq 0; \qquad s \in S,\ t \in T \cup \{\tau + 1\},\ l \in \{0\} \cup T$

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- $l = 0$: initial = 15
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- $l = 2$: qty entering = 00, exiting = 20

	t=1	t=2	t=3
ψ^0_s:	15	0	0
ψ^1_s:	5	20	5

Notation and Definitions

Example

$\psi^{l}_{s,t} \geq 0; \qquad s \in S,\ t \in T \cup \{\tau + 1\},\ l \in \{0\} \cup T$

$T = [1, 2]$

- $l = 0$: initial = 15
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Facility Location-based formulation for the lot sizing problem (Krarup and Bilde, 1977)

Notation and Definitions

Second-echelon

- Q^2: Second-echelon vehicle capacity
- K^2: number of 2E-vehicles available
- R: set of second-echelon routes
- W_r^t: set of route delivery patterns (RDPs) associated route r in period t
- w : RDP

Notation and Definitions

Route Delivery Pattern RDP

Consider a route r and a $w \in W_r^t$:

- $r = (s, \quad c_1, \quad c_2, \quad s)$
- $w = \quad (q_{wc_1}, \quad q_{wc_2})$
- $q_{wc} = (q_{wc}^t, q_{wc}^{t+1}, ..., q_{wc}^{\tau+1}) = (q_{wc}^h)_{t \leq h \leq \tau+1}$: Customer Delivery Pattern (CDP)

Desaulniers et al. (2016)

Notation and Definitions

Route Delivery Pattern RDP

- $q_{wc} = (q_{wc}^{t}, q_{wc}^{t+1}, \ldots, q_{wc}^{\tau+1}) = (q_{wc}^{h})_{t \leq h \leq \tau+1}$

FIFO Rule:

- $q_{wc}^{h} \in [0,\ UB_{tc}^{h}], \quad h \in T_{ct}^{+}$

Notation and Definitions

CDP

- Full subdelivery (F): $q_{wc}^{h} = UB_{tc}^{h}$
- Null subdelivery (Z): $q_{wc}^{h} = 0$
- Partial subdelivery (P)

Example: CDP = FPZ

Notation and Definitions

CDP

- Full subdelivery (F): $q^{h}_{wc} = UB^{h}_{tc}$
- Null subdelivery (Z): $q^{h}_{wc} = 0$
- Partial subdelivery (P)

RDP is a sequence of CDPs: e.g. $w = (FP, ZZ)$

Extreme RDPs: at most one partial subdelivery

Convex combination of extreme RDPs

Notation and Definitions

2nd Echelon Variables

- $\alpha^t_{r,w} \geq 0; \qquad r \in R,\ t \in T,\ w \in W^t_r$
- $\alpha^t_r = \sum_{w \in W^t_r} \alpha^t_{r,w} \in \{0, 1\}; \qquad r \in R,\ t \in T$

$$q_{wc} = \sum_{h \in T^+_{ct}} q^h_{wc}\, \alpha^t_{rw}$$

Problem Formulation

$$\min \sum_{t\in T}\left[\sum_{p\in P} f_p\,\lambda_p^t + \sum_{r\in R}\sum_{w\in W_r^t} f_{rw}\,\alpha_{r,w}^t + \sum_{s\in S}\sum_{l=0}^{t}\sum_{h=t+1}^{\tau+1} f_s^H \psi_{s,h}^l\right] \quad (1)$$

s.t.

Inventory constraints

$$\sum_{l=0}^{t} \psi_{s,t}^l = \sum_{r\in R_s}\sum_{w\in W_r^t} q_w\,\alpha_{r,w}^t \qquad \forall s\in S, t\in T \quad (2)$$

$$\sum_{t\in T_{ch}^-}\sum_{r\in R_c}\sum_{w\in W_r^t} q_{wc}^h \alpha_{rw}^t = \bar{d}_c^h \qquad \forall c\in N, h\in T \text{ such that } \bar{d}_c^h > 0 \quad (3)$$

Problem Formulation

Inventory capacity

$$\sum_{l=0}^{t} \sum_{k=t}^{\tau+1} \psi_{s,k}^{l} \leq C_s \qquad \forall s \in S, t \in T \qquad (4)$$

$$I_c^{0,h} + \sum_{t \in \Gamma_{ch}^{-}} \sum_{r \in R_c} \sum_{w \in W_r^t} \sum_{\substack{l \in T_{ct}^{+} \\ l > h}} q_{w,c}^{l}\, \alpha_{r,w}^{t} + d_c^h \leq C_c \qquad \forall c \in N, h \in T \qquad (5)$$

Vehicle capacity

$$\sum_{s \in S_p} \sum_{k=l}^{\tau+1} \psi_{s,k}^{l} \leq Q^1 + Q^1(|S_p| - 1)(1 - \lambda_p^l) \qquad \forall p \in P,\ l \in T \qquad (6)$$

$$\sum_{p \in P_s} \lambda_p^t \leq 1 \qquad \forall s \in S, t \in T \qquad (7)$$

Single visit

$$\sum_{r \in R_c} \sum_{w \in W_r^t} \alpha_{r,w}^{t} \leq 1 \qquad \forall c \in N,\ \forall t \in T \qquad (8)$$

Problem Formulation

Number of vehicles

$$\sum_{p \in P} \lambda_p^t \leq K^1 \qquad \forall t \in T \tag{9}$$

$$\sum_{r \in R} \sum_{w \in W_r^t} \alpha_{r,w}^t \leq K^2 \qquad \forall t \in T \tag{10}$$

Initial inventory

$$\sum_{t=1}^{\tau+1} \psi_{s,t}^0 = I_s^0 \qquad \forall s \in S \tag{11}$$

$$\sum_{t=l}^{\tau+1} \psi_{s,t}^l \leq Q^1 \sum_{p \in P_s} \lambda_p^l \qquad \forall s \in S, l \in T \tag{12}$$

Linking variables

$$\sum_{p \in P_s} \lambda_p^l \leq \sum_{t=l}^{\tau+1} \psi_{s,t}^l \qquad \forall s \in S, l \in T \tag{13}$$

Problem Formulation

Variable definition

$$\alpha_r^t = \sum_{w \in W_r^t} \alpha_{r,w}^t \qquad \forall r \in R,\ t \in T \tag{14}$$

Domain definition

$$\psi_{s,t}^l \geq 0 \qquad \forall s \in S,\ t \in T \cup \{\tau + 1\},\ 0 \leq l \leq t \tag{15}$$

$$\alpha_{r,w}^t \geq 0 \qquad \forall r \in R,\ t \in T,\ w \in W_r^t \tag{16}$$

$$\alpha_r^t \in \{0, 1\} \qquad \forall r \in R,\ t \in T \tag{17}$$

$$\lambda_p^t \in \{0, 1\} \qquad \forall p \in P,\ t \in T, \tag{18}$$

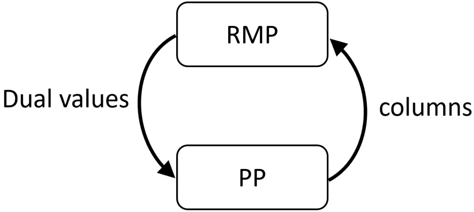
Branch-and-Price Algorithm

- First-echelon routes: enumeration
  - Only non-dominated routes
- Second-echelon routes: column generation

Branch-and-Price Algorithm

Column generation:

- Restricted Master Problem (RMP)
- Pricing Problem (PP)



Branch-and-Price Algorithm

- Pricing problem:
  - For each satellite and each period
  - Elementary shortest path problem with resource constraints (ESPPRC) combined with the linear relaxation of a knapsack problem

Labeling algorithm

Labeling Algorithm (Desaulniers et al., 2016)

- RDP information in the label for each customer
- Extension to each possible CDP
  - number of labels increase

Accelerating Techniques

- CDP handling
- Ng-path relaxation
- Bidirectional labeling
- Symmetry break

- Heuristic column generators
- Parallel computing

Branching

Branch on:

- first-echelon routes
- second-echelon routes

Integer solve of the RMP:

- root node
- after 20 nodes

Instances

400 instances derived from (Archetti et al., 2007)

- (supplier, satellite) $\in \{(1,2),(2,3)\}$
- up to 25 customers
- nb of second-echelon vehicles: $\{2,..,5\}$
- 3 time periods
- high and low inventory costs $(H3, L3)$

Solving time of 3 hours.

Results

Table: Average Gaps (in Percentage) per Instance Group and Number of Vehicles

Instance Class	Combination	Gap_0					Gap_{20}					Gap_f				
		K^2=2	K^2=3	K^2=4	K^2=5	Average	K^2=2	K^2=3	K^2=4	K^2=5	Average	K^2=2	K^2=3	K^2=4	K^2=5	Average
H3	1s2	57.70	51.12	47.11	44.04	49.84	7.71	7.65	6.37	5.64	6.72	10.05	6.31	4.71	3.89	6.16
	2s3	57.78	54.27	50.29	47.60	52.38	12.37	11.96	11.96	11.91	12.02	14.22	10.19	7.68	4.99	9.17
	Average H3	57.74	52.69	48.70	45.82	51.11	9.96	9.86	9.30	8.90	9.43	12.14	8.25	6.19	4.44	7.67
L3	1s2	63.27	55.18	50.51	45.58	53.33	6.85	7.24	6.91	6.05	6.71	10.42	6.21	4.85	3.70	6.17
	2s3	59.97	60.39	56.78	53.52	57.60	11.95	13.96	15.16	15.24	14.33	12.25	11.84	9.94	7.18	10.24
	Average L3	61.62	57.79	53.65	49.55	55.47	9.31	10.60	11.23	10.64	10.55	11.33	9.03	7.40	5.44	8.20
Average		59.64	55.24	51.17	47.69	53.27	9.64	10.25	10.26	9.79	10.00	11.74	8.64	6.80	4.94	7.93

Results

Table: Number of Instances by Integrality Gap Range and Instance Group

Instance Class	Combination	Optimal Solution	Gap$_f < 5\%$	Gap$_f \geq 5\%$	No Solution
H3	1s2	30	15	53	2
	2s3	27	13	58	2
	H3 Subtotal	57	28	111	4
L3	1s2	31	23	43	3
	2s3	28	9	60	3
	L3 Subtotal	59	32	103	6
Total		116	60	214	10

Conclusion

- Path-based formulation for the 2E-IRP
- Branch-and Price algorithm
- 116 optimal solutions
- An upper bound for 274 instances
- Extensions:
  - tightening the lower bound: cutting planes
  - direct visits, time windows, ...
  - heuristic

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Thank you for your attention!

Questions?

s.charaf@tue.nl