A Two-Echelon Inventory-Routing Problem

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Content

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- Problem Description
- Branch-and-Price Algorithm
- Results
- Conclusion
Motivation

- Many applications: city logistics, grocery distribution, e-commerce ...
- Last mile increasingly congested
- Sustainable transport
<table>
<thead>
<tr>
<th>Two-Echelon Vehicle Routing Problem</th>
<th>Perboli et al. (2011), Breunig et al. (2016), and Marques et al. (2020)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory-Routing Problem</td>
<td>Archetti et al. (2007), Guimarães et al. (2020), and Desaulniers et al. (2016)</td>
</tr>
<tr>
<td>Two-Echelon Inventory-Routing Problem</td>
<td>Rohmer et al. (2019) and Farias et al. (2021)</td>
</tr>
</tbody>
</table>
Problem Description

Legend

$U_1$, $U_2$

supplier (U)
Problem Description

A Two-Echelon Inventory-Routing Problem

Legend:
- Triangle (U): supplier (U)
- Square (S): satellite (S)
Problem Description

A Two-Echelon Inventory-Routing Problem

Legend
- △ supplier (U)
- □ satellite (S)
- ○ customer (C)

\( T = [1, \tau] \)  time horizon

\( d_{13} = [10, 10, 12] \) customer demand
Problem Description

- Vendor Managed Inventory
- Maximum level policy

Legend
- supplier (U)
- satellite (S)
- customer (C)
- inventory level $I_i^c$
- inventory capacity

$C_{S2} = 22$
$C_{13} = 22$
Problem Description

A Two-Echelon Inventory-Routing Problem

Legend
- supplier (U)
- satellite (S)
- customer (C)
- inventory level $I^t_i$
Problem Description

A Two-Echelon Inventory-Routing Problem

Legend
- \( U \) - supplier (U)
- \( S \) - satellite (S)
- \( C \) - customer (C)
- Green arrows - first-echelon route

Diagram showing a network with suppliers, satellites, and customers connected by routes.
Problem Description

Minimize transportation and inventory costs
Problem Description

Routing constraints:
(1) Vehicle capacity & (2) Nb of vehicles available
(3) Single visit to a customer/satellite
Problem Description

Inventory constraints:
(1) Inventory capacity & (2) no stock-out
Notation and Definitions

First-echelon

- $Q^1$: first-echelon vehicle capacity
- $K^1$: number of 1E-vehicles available
- $P$: set of first-echelon routes

Variables

- $\lambda^t_p \in \{0, 1\}; \ p \in P, \ t \in T$
- $\psi^l_{s,t} \geq 0; \ s \in S, \ t \in T \cup \{\tau + 1\}, \ l \in \{0\} \cup T$
Example

\[ \psi^l_{s,t} \geq 0; \quad s \in S, \ t \in T \cup \{\tau + 1\}, \ l \in \{0\} \cup T \]

\[ T = [1, 2] \]

- \( l = 0 \): initial = 15
- \( l = 1 \): qty entering = 30, exiting = 20
- \( l = 2 \): qty entering = 00, exiting = 20
Notation and Definitions

Example

\[ \psi_{s,t}^l \geq 0; \quad s \in S, \ t \in T \cup \{\tau + 1\}, \ l \in \{0\} \cup T \]

\[ T = [1, 2] \]

- \( l = 0 \): initial = 15
- \( l = 1 \): qty entering = 30, exiting = 20
- \( l = 2 \): qty entering = 00, exiting = 20

\[
\begin{array}{c|c|c|c}
\psi_s^0 & t=1 & t=2 & t=3 \\
\hline
15 & 0 & 0 \\
\hline
\psi_s^1 & 5 & 20 & 5 \\
\end{array}
\]
Notation and Definitions

Example

\[ \psi_{s,t}^l \geq 0; \quad s \in S, \ t \in T \cup \{\tau + 1\}, \ l \in \{0\} \cup T \]

\[ T = [1, 2] \]

- \( l = 0 \): initial = 15
- \( l = 1 \): qty entering = 30, exiting = 20
- \( l = 2 \): qty entering = 00, exiting = 20

Facility Location-based formulation for the lot sizing problem (Krarup and Bilde, 1977)
## Notation and Definitions

### Second-echelon

- $Q^2$: Second-echelon vehicle capacity
- $K^2$: number of 2E-vehicles available
- $R$: set of second-echelon routes
- $W_r^t$: set of route delivery patterns (RDPs) associated route $r$ in period $t$
- $w$: RDP
Notation and Definitions

Route Delivery Pattern RDP

Consider a route \( r \) and a \( w \in W_r^t \):

- \( r = (s, c_1, c_2, s) \)
- \( w = (q_{w1c}, q_{w2c}) \)
- \( q_{wc} = (q_{w1c}^t, q_{w1c}^{t+1}, \ldots, q_{w1c}^{\tau+1}) = (q_{w1c}^h)_{t \leq h \leq \tau+1} \) : Customer Delivery Pattern (CDP)

Desaulniers et al. (2016)
Notation and Definitions

Route Delivery Pattern RDP

- \( q_{wc} = (q^t_{wc}, q^{t+1}_{wc}, \ldots, q^{\tau+1}_{wc}) = (q^h_{wc})_{t \leq h \leq \tau + 1} \)

FIFO Rule:

- \( q^h_{wc} \in [0, UB^h_{tc}], \quad h \in T^+_ct \)
Notation and Definitions

CDP

- Full subdelivery (F): \( q_{wc}^h = UB_{tc}^h \)
- Null subdelivery (Z): \( q_{wc}^h = 0 \)
- Partial subdelivery (P)

Example: CDP = FPZ
Notation and Definitions

CDP

- Full subdelivery (F): $q_{wc}^h = UB_{tc}^h$
- Null subdelivery (Z): $q_{wc}^h = 0$
- Partial subdelivery (P)

RDP is a sequence of CDPs: e.g. $w = (FP, ZZ)$

Extreme RDPs: at most one partial subdelivery

Convex combination of extreme RDPs
Notation and Definitions

2nd Echelon Variables

- $\alpha^t_{r,w} \geq 0$; $r \in R$, $t \in T$, $w \in W^t_r$
- $\alpha^t_r = \sum_{w \in W^t_r} \alpha^t_{r,w} \in \{0, 1\}$; $r \in R$, $t \in T$

$q_{wc} = \sum_{h \in T^+_c t} q^h_{wc} \alpha^t_{rw}$
Problem Formulation

\[
\begin{align*}
\min \sum_{t \in T} \left[ \sum_{p \in P} f_p \lambda_p^t + \sum_{r \in R} \sum_{w \in W_t^r} f_{rw} \alpha_{r,w}^t + \sum_{s \in S} \sum_{l=0}^{t} \sum_{h=t+1}^{\tau+1} f_s^h \psi_{s,h}^l \right]
\end{align*}
\]  \hspace{1cm} (1)

s.t.

\[
\sum_{l=0}^{t} \psi_{s,t}^l = \sum_{r \in R_s} \sum_{w \in W_t^r} q_w \alpha_{r,w}^t \hspace{1cm} \forall s \in S, t \in T
\]  \hspace{1cm} (2)

\[
\sum_{t \in T^-_{ch}} \sum_{r \in R_c} \sum_{w \in W_t^r} q_{wc}^h \alpha_{rw}^t = \bar{d}_c^h \hspace{1cm} \forall c \in N, h \in T \text{ such that } \bar{d}_c^h > 0
\]  \hspace{1cm} (3)

Inventory constraints
Problem Formulation

**Inventory Capacity**

\[
\sum_{t=0}^{\tau+1} \sum_{k=t}^{l} \psi_{s,k}^{l} \leq C_s \quad \forall s \in S, t \in T \quad (4)
\]

\[
l_{c}^{0,h} + \sum_{t \in \Gamma_{ch}^-} \sum_{r \in R_c} \sum_{w \in W_{r}^l} \sum_{l \in T_{ct}^+}^{l > h} q_{w,c}^{l} \alpha_{r,w}^{t} + d_{c}^{h} \leq C_c \quad \forall c \in N, h \in T \quad (5)
\]

**Vehicle Capacity**

\[
\sum_{s \in S_p}^{\tau+1} \sum_{k=s}^{l} \psi_{s,k}^{l} \leq Q^1 + Q^1(|S_p| - 1)(1 - \lambda_{p}^{l}) \quad \forall p \in P, l \in T \quad (6)
\]

\[
\sum_{p \in P_{s}} \lambda_{p}^{t} \leq 1 \quad \forall s \in S, t \in T \quad (7)
\]

**Single Visit**

\[
\sum_{r \in R_c} \sum_{w \in W_{r}^t} \alpha_{r,w}^{t} \leq 1 \quad \forall c \in N, \forall t \in T \quad (8)
\]
Problem Formulation

Number of vehicles

\[ \sum_{p \in P} \lambda^t_p \leq K^1 \quad \forall t \in T \quad (9) \]

\[ \sum_{r \in R} \sum_{w \in W^t_r} \alpha^t_{r,w} \leq K^2 \quad \forall t \in T \quad (10) \]

Initial inventory

\[ \sum_{t=1}^{\tau+1} \psi^0_{s,t} = l^0_s \quad \forall s \in S \quad (11) \]

\[ \sum_{t=l}^{\tau+1} \psi^l_{s,t} \leq Q^1 \sum_{p \in P_s} \lambda^l_p \quad \forall s \in S, l \in T \quad (12) \]

Linking variables

\[ \sum_{p \in P_s} \lambda^l_p \leq \sum_{t=l}^{\tau+1} \psi^l_{s,t} \quad \forall s \in S, l \in T \quad (13) \]
# Problem Formulation

<table>
<thead>
<tr>
<th>Variable definition</th>
<th>$\alpha_r^t = \sum_{w \in W^t_r} \alpha_{r,w}^t$</th>
<th>$\forall r \in R, \ t \in T$</th>
<th>(14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain definition</td>
<td>$\psi_{s,t}^l \geq 0$</td>
<td>$\forall s \in S, \ t \in T \cup {\tau + 1}, \ 0 \leq l \leq t$</td>
<td>(15)</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{r,w}^t \geq 0$</td>
<td>$\forall r \in R, \ t \in T, \ w \in W^t_r$</td>
<td>(16)</td>
</tr>
<tr>
<td></td>
<td>$\alpha_r^t \in {0, 1}$</td>
<td>$\forall r \in R, \ t \in T$</td>
<td>(17)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_p^t \in {0, 1}$</td>
<td>$\forall p \in P, \ t \in T$,</td>
<td>(18)</td>
</tr>
</tbody>
</table>
Branch-and-Price Algorithm

- First-echelon routes: enumeration
  - Only non-dominated routes

- Second-echelon routes: column generation
Branch-and-Price Algorithm

Column generation:

- Restricted Master Problem (RMP)
- Pricing Problem (PP)
Branch-and-Price Algorithm

Pricing problem:
- For each satellite and each period
- Elementary shortest path problem with resource constraints (ESPPRC) combined with the linear relaxation of a knapsack problem
Labeling Algorithm (Desaulniers et al., 2016)

- RDP information in the label for each customer
- Extension to each possible CDP
  - number of labels increase
Accelerating Techniques

- CDP handling
- Ng-path relaxation
- Bidirectional labeling
- Symmetry break
- Heuristic column generators
- Parallel computing
Branching

Branch on:

- first-echelon routes
- second-echelon routes

Integer solve of the RMP:

- root node
- after 20 nodes
Instances

400 instances derived from (Archetti et al., 2007)

- (supplier, satellite) ∈ {(1, 2), (2, 3)}
- up to 25 customers
- nb of second-echelon vehicles: {2, .., 5}
- 3 time periods
- high and low inventory costs (H₃, L₃)

Solving time of 3 hours.
# Results

## Table: Average Gaps (in Percentage) per Instance Group and Number of Vehicles

<table>
<thead>
<tr>
<th>Instance Class</th>
<th>Combination</th>
<th>$K^2=2$</th>
<th>$K^2=3$</th>
<th>$K^2=4$</th>
<th>$K^2=5$</th>
<th>Average</th>
<th>$K^2=2$</th>
<th>$K^2=3$</th>
<th>$K^2=4$</th>
<th>$K^2=5$</th>
<th>Average</th>
<th>$K^2=2$</th>
<th>$K^2=3$</th>
<th>$K^2=4$</th>
<th>$K^2=5$</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>H3 1s2</td>
<td>57.70</td>
<td>51.12</td>
<td>47.11</td>
<td>44.04</td>
<td>49.84</td>
<td></td>
<td>7.71</td>
<td>7.65</td>
<td>6.37</td>
<td>5.64</td>
<td>6.72</td>
<td></td>
<td>10.05</td>
<td>6.31</td>
<td>4.71</td>
<td>3.89</td>
</tr>
<tr>
<td>2s3</td>
<td>57.78</td>
<td>54.27</td>
<td>50.29</td>
<td>47.60</td>
<td>52.38</td>
<td></td>
<td>12.37</td>
<td>11.96</td>
<td>11.96</td>
<td>11.91</td>
<td>12.02</td>
<td></td>
<td>14.22</td>
<td>10.19</td>
<td>7.68</td>
<td>4.99</td>
</tr>
<tr>
<td>Average H3</td>
<td>57.74</td>
<td>52.69</td>
<td>48.70</td>
<td>45.82</td>
<td>51.11</td>
<td></td>
<td>9.96</td>
<td>9.86</td>
<td>9.30</td>
<td>8.90</td>
<td>9.43</td>
<td></td>
<td>12.14</td>
<td>8.25</td>
<td>6.19</td>
<td>4.44</td>
</tr>
<tr>
<td>L3 1s2</td>
<td>63.27</td>
<td>55.18</td>
<td>50.51</td>
<td>45.58</td>
<td>53.33</td>
<td></td>
<td>6.85</td>
<td>7.24</td>
<td>6.91</td>
<td>6.05</td>
<td>6.71</td>
<td></td>
<td>10.42</td>
<td>6.21</td>
<td>4.85</td>
<td>3.70</td>
</tr>
<tr>
<td>2s3</td>
<td>59.97</td>
<td>60.39</td>
<td>56.78</td>
<td>53.52</td>
<td>57.60</td>
<td></td>
<td>11.95</td>
<td>13.96</td>
<td>15.16</td>
<td>15.24</td>
<td>14.33</td>
<td></td>
<td>12.25</td>
<td>11.84</td>
<td>9.94</td>
<td>7.18</td>
</tr>
<tr>
<td>Average L3</td>
<td>61.62</td>
<td>57.79</td>
<td>53.65</td>
<td>49.55</td>
<td>55.47</td>
<td></td>
<td>9.31</td>
<td>10.60</td>
<td>11.23</td>
<td>10.64</td>
<td>10.55</td>
<td></td>
<td>11.33</td>
<td>9.03</td>
<td>7.40</td>
<td>5.44</td>
</tr>
<tr>
<td>Average</td>
<td>59.64</td>
<td>55.24</td>
<td>51.17</td>
<td>47.69</td>
<td>53.27</td>
<td></td>
<td>9.64</td>
<td>10.25</td>
<td>10.26</td>
<td>9.79</td>
<td>10.00</td>
<td></td>
<td>11.74</td>
<td>8.64</td>
<td>6.80</td>
<td>4.94</td>
</tr>
</tbody>
</table>
## Results

### Table: Number of Instances by Integrality Gap Range and Instance Group

<table>
<thead>
<tr>
<th>Instance Class</th>
<th>Combination</th>
<th>Optimal Solution</th>
<th>$\text{Gap}_f &lt; 5%$</th>
<th>$\text{Gap}_f \geq 5%$</th>
<th>No Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>H3</td>
<td>1s2</td>
<td>30</td>
<td>15</td>
<td>53</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2s3</td>
<td>27</td>
<td>13</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>H3 Subtotal</td>
<td>57</td>
<td>28</td>
<td>111</td>
<td>4</td>
</tr>
<tr>
<td>L3</td>
<td>1s2</td>
<td>31</td>
<td>23</td>
<td>43</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2s3</td>
<td>28</td>
<td>9</td>
<td>60</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>L3 Subtotal</td>
<td>59</td>
<td>32</td>
<td>103</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>116</td>
<td>60</td>
<td>214</td>
<td>10</td>
</tr>
</tbody>
</table>
Conclusion

- Path-based formulation for the 2E-IRP
- Branch-and Price algorithm
- 116 optimal solutions
- An upper bound for 274 instances
- Extensions:
  - tightening the lower bound: cutting planes
  - direct visits, time windows, ...
  - heuristic


Thank you for your attention!

Questions?

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